Geometry Constrained Weakly Supervised Object Localization

Weizeng Lu[1,2], Xi Jia[3], Weicheng Xie[1,2], Linlin Shen[1,2](✉), Yicong Zhou[4], and Jinming Duan[3](✉)

[1] Computer Vision Institute, School of Computer Science and Software Engineering, Shenzhen University, China
[2] Shenzhen Institute of Artificial Intelligence and Robotics for Society, China
[3] School of Computer Science, University of Birmingham, United Kingdom
[4] Department of Computer and Information Science, University of Macau, China
luweizeng2018@email.szu.edu.cn, wcxie@szu.edu.cn, llshen@szu.edu.cn, x.jia.1@cs.bham.ac.uk, yicongzhou@um.edu.mo, j.duan@cs.bham.ac.uk

1 Mathematical Models of Different Geometric Shapes

In section 3.2 of the main paper, we have defined the mathematical model of ellipse for the model-driven mask generator. We introduce here the mathematical models for three geometric shapes (i.e. rectangle, rotated rectangle, and rotated ellipse) in detail.

Rotated ellipse: Given the coefficients (c_x, c_y, θ, a, b) of a rotated ellipse, the mathematical model of a rotated ellipse can be defined as

$$\phi(x,y) = \frac{((x-c_x)cos\theta + (y-c_y)sin\theta)^2}{a^2} + \frac{((x-c_x)sin\theta - (y-c_y)cos\theta)^2}{b^2} - 1, \tag{1}$$

where $x, y : \Omega \subset \mathbb{R}^2$.

Rectangle: Given the coefficients (c_x, c_y, a, b) of a rectangle, we can represent a rectangle with the mathematical model defined as below

$$\phi(x,y) = \left|\frac{x-c_x}{a} + \frac{y-c_y}{b}\right| + \left|\frac{x-c_x}{a} - \frac{y-c_y}{b}\right| - 1. \tag{2}$$

Rotated rectangle: Given the coefficients (c_x, c_y, θ, a, b) of a rotated rectangle, the mathematical model of a rotated rectangle can be defined as

$$\begin{aligned}\phi(x,y) = &\left|\frac{(x-c_x)cos\theta - (y-c_y)sin\theta}{a} + \frac{(x-c_x)sin\theta + (y-c_y)cos\theta}{b}\right| + \\ &\left|\frac{(x-c_x)cos\theta - (y-c_y)sin\theta}{a} - \frac{(x-c_x)sin\theta + (y-c_y)cos\theta}{b}\right| - 1.\end{aligned} \tag{3}$$

The inverse of the tangent function to approximate the Heaviside function, the model-driven generator can be defined as:

$$H_\epsilon(\phi(x,y)) = \frac{1}{2}\left(1 + \frac{2}{\pi} arctan\left(\frac{\phi(x,y)}{\epsilon}\right)\right). \tag{4}$$

2 Derivatives w.r.t Shape Parameters

Since $M = H_\epsilon(\phi(x,y))$, the derivatives of M with respect to (w.r.t.) the parameters of a geometric shape can be transformed to those of ϕ. The derivative of $M_{x,y}$ w.r.t. the parameter ϵ can be calculated as follows

$$\frac{\partial M_{x,y}}{\partial \epsilon} = \frac{1}{\pi} \frac{1}{1 + (\frac{\phi(x,y)}{\epsilon})^2} \frac{-\phi(x,y)}{\epsilon^2}. \tag{5}$$

We take the parameter a from detector outputs (i.e. c_x, c_y, θ, a, b) as an example to introduce the gradient transfer of generator for updating detector parameters, the derivatives of parameter a, i.e. $\frac{\partial M_{x,y}}{\partial a}$,are calculated as follows

$$\frac{\partial M_{x,y}}{\partial a} = \frac{1}{\pi} \frac{1}{1 + (\frac{\phi(x,y)}{\epsilon})^2} \frac{\partial \phi(x,y)}{\partial a}, \tag{6}$$

For the shape of **Rotated ellipse**, the derivative $\frac{\partial \phi(x,y)}{\partial a}$ is easily to calculate as follows

$$\frac{\partial \phi(x,y)}{\partial a} = -\frac{2((x-c_x)\cos\theta + (y-c_y)\sin\theta)^2}{a^3}, \tag{7}$$

while $\frac{\partial M_{x,y}}{\partial c_x}$, $\frac{\partial M_{x,y}}{\partial c_y}$, $\frac{\partial M_{x,y}}{\partial b}$ and $\frac{\partial M_{x,y}}{\partial \theta}$ are derived similarly as $\frac{\partial M_{x,y}}{\partial a}$.

For the shape of **Rectangle**, we denote $\alpha = \alpha(c_x, a) \doteq \frac{x-c_x}{a}$ and $\beta = \beta(c_y, b) \doteq \frac{y-c_y}{b}$. To obtain the derivative of $\phi(x,y)$ w.r.t. the four parameters, i.e. w,h,c_x,c_y, in Eq. (2), then the derivatives of ϕ w.r.t. the four parameters can be transformed those w.r.t. α and β as follows

$$\frac{\partial \phi}{\partial a} = \frac{\partial \phi}{\partial \alpha} \frac{\partial \alpha}{\partial a}, \tag{8}$$

where the terms alike $\frac{\partial \alpha}{\partial a}$ are easy to derive. While the sub-gradient of $|x|$ w.r.t. x is zero at the point $x = 0$, the derivative of $\frac{\partial \phi}{\partial \alpha}$ is obtained as follows

$$\frac{\partial \phi}{\partial \alpha} = \begin{cases} 2 & \text{if } \alpha > |\beta|, \\ 1 & \text{if } \alpha = |\beta| > 0, \\ 0 & \text{if } |\alpha| < |\beta| \text{ or } \alpha = \beta = 0. \\ -1 & \text{if } \alpha = -|\beta| < 0, \\ -2 & \text{if } \alpha < -|\beta|, \end{cases} \tag{9}$$

the derivative of $\frac{\partial \phi}{\partial \beta}$ can be similarly obtained.

For the shape of **Rotated Rectangle**, we denote $\alpha = \alpha(c_x, c_y, a, \theta) \doteq \frac{(x-c_x)cos\theta-(y-c_y)sin\theta}{a}$ and $\beta = \beta(c_y, c_y, b, \theta) \doteq \frac{(x-c_x)sin\theta+(y-c_y)cos\theta}{b}$. The similar derivatives as Eq. (8) are derived as follows

$$\begin{cases} \frac{\partial \phi}{\partial a} = \frac{\partial \phi}{\partial \alpha} \frac{\partial \alpha}{\partial a}, \\ \frac{\partial \phi}{\partial c_x} = \frac{\partial \phi}{\partial \alpha} \frac{\partial \alpha}{\partial c_x} + \frac{\partial \phi}{\partial \beta} \frac{\partial \beta}{\partial c_x}, \\ \frac{\partial \phi}{\partial \theta} = \frac{\partial \phi}{\partial \alpha} \frac{\partial \alpha}{\partial \theta} + \frac{\partial \phi}{\partial \beta} \frac{\partial \beta}{\partial \theta}. \end{cases} \tag{10}$$

where the derivative of $\frac{\partial \phi}{\partial \alpha}$ is the same as that in Eq. (9).