Appendix: ProxyBNN: Learning Binarized Neural Networks via Proxy Matrices

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1 Appendix

1.1 The derivative to $Z_{i,k}$

Since the binary mapping involves all elements in a filter, the gradient of $Z_{i,k}$ requires the summation over $i = 1, \cdots, hwc$. That is

$$\frac{\partial \ell}{\partial Z_{i,k}} = \sum_{j=1}^{hwc} \frac{\partial \ell}{\partial \psi(Z_i)_j} \frac{\partial \psi(Z_i)_j}{Z_{i,k}} \tag{1}$$

$$= \sum_{j=1}^{hwc} \frac{\partial \ell}{\partial \psi(Z_i)_j} \frac{\partial\big(\frac{\sum_{n=1}^{hwc}|Z_{i,n}|}{hwc} sgn(Z_{i,j})\big)}{\partial Z_{i,k}} \tag{2}$$

$$= \sum_{j=1}^{hwc} \frac{\partial \ell}{\partial \psi(Z_i)_j} \Big(\frac{sgn(Z_{i,j})}{hwc} sgn(Z_{i,k}) + \frac{\sum_{n=1}^{hwc}|Z_{i,n}|}{hwc} \frac{\partial sgn(Z_{i,j})}{\partial Z_{i,k}}\Big) \tag{3}$$

$$= \frac{sgn(Z_{i,k})}{hwc} \sum_{j=1}^{hwc} \frac{\partial \ell}{\partial \psi(Z_i)_j} sgn(Z_{i,j}) + \frac{\partial \ell}{\partial \psi(Z_i)_k} \frac{\sum_{n=1}^{hwc}|Z_{i,n}|}{hwc} \frac{\partial sgn(Z_{i,k})}{\partial Z_{i,k}}. \tag{4}$$

By solving the constrained optimization problem in (3), we have

$$Z_{i,k} \approx \alpha_i B_{i,k} \approx \psi(Z_i)_k. \tag{5}$$

Considering an infinitesimal change in the above equation with respect to $Z_{i,k}$, we obtain

$$\mathrm{d}Z_{i,k} \approx \mathrm{d}\frac{\sum_{j=1}^{hwc}|Z_{i,j}|}{hwc} sgn(Z_{i,k}) + \frac{\sum_{j=1}^{hwc}|Z_{i,j}|}{hwc} \mathrm{d}sgn(Z_{i,k}), \tag{6}$$

which can be further simplified as

$$\mathrm{d}sgn(Z_{i,k}) \approx (1 - \frac{1}{hwc}) \frac{hwc}{\sum_{j=1}^{hwc}|Z_{i,j}|} \mathrm{d}Z_{i,k} \approx \frac{hwc}{\sum_{j=1}^{hwc}|Z_{i,j}|} \mathrm{d}Z_{i,k}. \tag{7}$$

Combining (4) and (7), we define the derivative to $Z_{i,k}$ as

$$\frac{\partial \ell}{\partial Z_{i,k}} := \frac{sgn(Z_{i,k})}{h \cdot w \cdot c} \sum_{j=1}^{hwc} \frac{\partial \ell}{\partial \psi(Z_i)_j} sgn(Z_{i,j}) + \frac{\partial \ell}{\partial \psi(Z_i)_k}. \tag{8}$$