A Path Constraints

Lemma 2 (Paths Constraints). *Let $G' = (V', E', w) = (V \cup T, E \cup E^S, w)$ be an edge-weighted graph extended by terminal nodes T. For any edge indicator $y \in 0, 1^{|E \cup S|}$ that satisfies*

$$\sum_{t \in T} y_{tv} = |T| - 1 \,\forall v \in V \tag{36}$$

the following set of constraints are equivalent:

$$\begin{aligned} & y_{ut} + y_{uv} + y_{vt'} \geq 1 && \forall (u, v) \in E \ \forall t, t' \in T, \, t \neq t' && (37)\\ \Leftrightarrow & y_{tu} + y_{uv} \geq y_{tv}, && \forall uv \in E, t \in T && (38)\\ & y_{tv} + y_{uv} \geq y_{tu}, && \forall uv \in E, t \in T. && (39) \end{aligned}$$

Proof. This lemma is trivially fulfilled for $|T| \leq 1$. We will prove the lemma for $|T| > 1$ by contradiction in each direction.

"⇒" Assume eq. (37) holds and $\exists y_{tv} > y_{tu} + y_{uv}$. In case $y_{tv} = 1$, eq. (36) implies that $\exists t' \neq t : y_{tv} = 0$ which leads to the contradiction

$$y_{tv} > y_{tu} + y_{uv} \geq 1 - y_{t'u} = 1 \tag{40}$$

In case $y_{tv} = 0$, eq. (36) implies that $\forall t' \neq t : y_{t'v} = 0$ leading to the contradiction

$$y_{tv} > y_{tu} + y_{uv} \geq 1 - y_{t'u} = 0. \tag{41}$$

The proof for eq. (39) is analogous.

"⇐" Assume eqs. (38) and (39) hold and $\exists (u, v) \in E \, t, t' \in T, \, t \neq t' : y_{ut} + y_{uv} + y_{vt'} < 1$. This leads to the contradiction

$$\begin{aligned} & y_{ut} + y_{uv} + y_{vt'} < 1 && (42)\\ \Rightarrow & y_{ut} = 0, y_{uv} = 0 \text{ and } \underbrace{y_{vt'} = 0}_{\Rightarrow y_{vt} = 1} && (43)\\ \Rightarrow & 1 = y_{vt} \geq y_{ut} + y_{uv} = 0. && (44) \end{aligned}$$