Rel Pose Est of Calibrated Cameras with Known SE(3) Invariants

Supplementary Material

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A Seven Cubics from 3P-RA-ST0 under NullE (NullEx)

Theorem 1. *Let $E = [t]_\times R$ be an essential matrix with R being a rotation around a vector u. If $u^\top t = 0$, then the following equations hold:*

$$(A_{ki}E_{ii}E_{jk} + A_{ij}E_{kk}E_{ii} - A_{jk}(E_{kk}E_{ik} - E_{ki}E_{jj} + E_{kj}E_{ji} + E_{ij}E_{jk}))\tau' + 2A_{ij}A_{jk}^2 = 0, \quad (23)$$

$$(A_{12}A_{23}E_{31}+A_{12}A_{31}E_{32}+A_{12}E_{12}E_{33}+A_{23}A_{31}E_{21}+A_{23}E_{11}E_{23}+A_{31}E_{22}E_{31})\tau' + 2A_{12}A_{23}A_{31} = 0, \quad (24)$$

where $\tau' = \operatorname{tr} R + 1$, $A = E - E^\top$, and E_{ij} (resp. A_{ij}) are the entries of matrix E (resp. A). The indices i, j, k are intended to be different in Eq. (23)*.*

Proof. By a straightforward computation. Constraints (23) and (24) are derived by using the implicitization algorithm [5]. First, assuming that R is represented by formula (1), we constructed the polynomial ideal J generated by $E_{ij} - ([t]_\times R)_{ij}$, $u^\top t$ and $\|u\|^2 + \sigma^2 - 1$. Then we computed the Gröbner basis of J with respect to a lexicographic ordering where the entries of vectors t and u are greater than the entries of matrix E and scalar σ. Thus we got the elements of the Gröbner basis not involving vectors t and u, i.e. the basis of the elimination ideal $J \cap \mathbb{C}[E_{11}, \ldots, E_{33}, \sigma]$. This basis contains all polynomials from (18) – (21) as well as seven more elements represented by (23) and (24). □

B 3P-RA-ST0: Solver Details

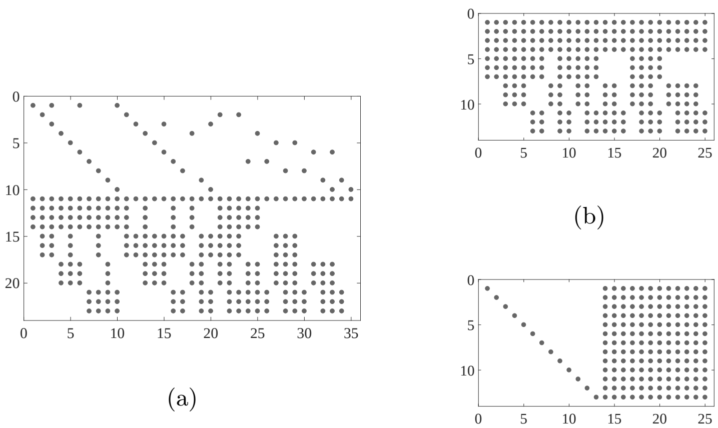
Recall that in Subsect. 6.3 we reduced solving the 3P-RA-ST0 problem to finding real roots of the polynomial system $Ax = 0_{23\times 1}$, where A is a coefficient matrix of size 23×35 and x is a monomial vector. The structure of matrix A is shown in Fig. 5(a). It can be seen that A has the following block form

$$A = \begin{bmatrix} U & V \\ W & X \end{bmatrix},$$

where U is an upper-triangular 10×10 matrix with 1's on its main diagonal. Then it follows that $\det U = 1$ and the inverse to U always exists. By elementary row operations, matrix A is equivalent to

$$\begin{bmatrix} U & V \\ 0_{13\times 10} & B \end{bmatrix},$$

where matrix $B = X - WU^{-1}V$ of size 13×25 contains all necessary data for computing all solutions of the initial polynomial system. The structure of matrix B is shown in Fig. 5(b).



(a) (b) (c)

Fig. 5. Left: The sparse structure of 23×35 matrix A. The gray circles represent non-zero entries. The 10×10 left upper submatrix of A is upper-triangular and its main diagonal consists of 1's. Right top: The final 13×25 elimination template B for computing the 12-th degree univariate polynomial. Right bottom: The reduced row echelon form of B

It is worth mentioning that matrix B should not be computed by its definition in an implementation. Instead, it is more efficient to use quite simple pre-computed formulas for the nonzero entries of B.

The initial polynomial system $Ax = 0_{23\times 1}$ is equivalent to the system $By = 0_{13\times 1}$, where y is a new monomial vector consisting of 25 monomials. We assume that the hidden variable is γ. Let the last 17 monomials in y be

$$\beta^2\gamma, \beta^2, \alpha\beta\gamma^2, \alpha\beta\gamma, \alpha\beta, \alpha\gamma^2, \alpha\gamma, \alpha, \beta\gamma^3, \beta\gamma^2, \beta\gamma, \beta, \gamma^4, \gamma^3, \gamma^2, \gamma, 1.$$

Let $\widetilde{B}$ be the reduced row echelon form of B, see Fig. 5(c). If $(\widetilde{B})_i$ is the i-th row of $\widetilde{B}$, then we can write

$$\begin{bmatrix} \gamma(\widetilde{B})_{13} - (\widetilde{B})_{12} \\ \gamma(\widetilde{B})_{12} - (\widetilde{B})_{11} \\ \gamma(\widetilde{B})_{10} - (\widetilde{B})_{9} \end{bmatrix} y = C(\gamma) \begin{bmatrix} \alpha \\ \beta \\ 1 \end{bmatrix} = 0_{3\times 1}.$$

Here we defined the 3×3 matrix

$$C(\gamma) = \begin{bmatrix} [3] & [4] & [5] \\ [3] & [4] & [5] \\ [3] & [4] & [5] \end{bmatrix},$$

where $[n]$ means a univariate polynomial in γ of degree n. It follows that γ is a root if and only if matrix $C(\gamma)$ is degenerate. The problem is thus converted to finding all real roots of the 12-th degree polynomial $p(\gamma) = \det C(\gamma)$. Let γ_0 be a real root of p. The remaining components of vector u, i.e. α and β, are computed from the right null-vector of matrix $C(\gamma_0)$. Hence we get all solutions for vector u.

It is important to note that because of numerical inaccuracy, the solution for u does not exactly satisfy Eq. (2). In order to rectify the solution, we replace u with the vector

$$\hat{u} = \frac{\sqrt{1-\sigma^2}}{\|u\|}\, u.$$

Then rotation matrix R is computed from the unit quaternion $\begin{bmatrix} \sigma & \hat{u}^\top \end{bmatrix}$ by formula (1).

Using the rigid motion ambiguity of the world coordinate frame, we set $t' = 0_{3\times 1}$. The translation vector t'' is found from the epipolar constraints as the right null-vector of the matrix

$$\begin{bmatrix} {q'_1}^\top R^\top [q''_1]_\times \\ {q'_2}^\top R^\top [q''_2]_\times \\ {q'_3}^\top R^\top [q''_3]_\times \end{bmatrix}.$$

The scale ambiguity allows us to set $\|t''\| = 1$. Finally, the sign of t'' is disambiguated by means of the cheirality constraint [11,30].

C Real World Data Performance

Table 1 and 2 corresponds to the real world data experiment performance discussed in Subsect. 7.3.

Dataset	%	3P-RA-ST0		4P-RA		4P-ST0	5P
		Odo	IMU	Odo	IMU		
RSeeds 0226b	25	8.9	**5.6**	10.3	6.7	8.8	10.8
	50	16.0	**9.4**	16.0	10.5	16.1	17.5
	75	26.9	**15.4**	29.1	15.7	29.1	27.7
RSeeds 0226a	25	7.3	**4.5**	10.5	6.7	7.0	9.1
	50	14.6	**7.7**	18.7	11.0	12.5	15.1
	75	27.4	**11.5**	33.7	15.7	21.8	23.6
RSeeds 0225b	25	10.1	**5.5**	10.7	7.5	8.8	10.6
	50	17.4	**9.2**	17.1	11.2	16.4	17.8
	75	35.9	**16.2**	34.7	16.8	30.8	30.0
RSeeds 0225a	25	9.1	**4.5**	11.5	6.6	6.9	9.8
	50	17.8	**8.3**	20.4	11.8	13.3	15.8
	75	32.6	**14.0**	39.3	17.4	22.8	24.9

Dataset	%	4P-ST0	5P	Dataset	%	4P-ST0	5P
TUM	25	**3.7**	4.1	TUM	25	**2.7**	3.8
RGBD	50	**6.4**	6.6	RGBD	50	**6.2**	7.2
#360	75	**12.1**	14.0	#1	75	**14.9**	15.2
TUM	25	**2.9**	4.3	TUM	25	**2.7**	3.1
RGBD	50	**5.7**	7.7	RGBD	50	**5.9**	6.2
#2	75	16.0	**15.2**	#3	75	13.2	**12.3**
R@H	25	**5.1**	7.9	R@H	25	**8.7**	9.2
anto	50	**10.3**	14.3	anto	50	**14.3**	21.5
s1 FC	75	**15.7**	22.8	s1 LC	75	**22.9**	39.3
R@H	25	**3.4**	7.2	R@H	25	**8.1**	8.5
alma	50	**9.3**	12.0	alma	50	**13.3**	15.4
s1 FC	75	**16.3**	16.9	s1 LC	75	**22.4**	24.8
R@H	25	**4.9**	5.3	R@H	25	**5.0**	6.4
pare	50	**8.5**	9.9	pare	50	**8.6**	11.7
s1 FC	75	**16.1**	23.5	s1 LC	75	**13.7**	21.5
R@H	25	**7.3**	9.0	R@H	25	**9.4**	12.9
sarmis	50	**12.2**	17.4	sarmis	50	**17.1**	29.8
s1 FC	75	**21.4**	28.0	s1 LC	75	**34.7**	46.9

Table 1. Translation error angle (deg) quantiles on indoor real data. Adding SE(3) invariant measurements improves relative pose estimation in most cases

Dataset	%	3P-...	4P-RA	4P-ST0	5P	Dataset	%	3P-...	4P-RA	4P-ST0	5P
UMich #1 FC	25	2.3	3.1	**2.0**	2.2	KITTI #4	25	0.9	0.9	**0.8**	0.9
	50	5.1	7.7	**4.2**	4.6		50	1.4	1.6	**1.2**	1.6
	75	14.2	37.0	**9.6**	9.9		75	2.4	2.9	**2.1**	2.6
UMich #2 FC	25	1.7	1.7	**1.5**	1.6	KITTI #6	25	0.9	1.2	**0.8**	1.0
	50	3.0	3.0	**2.8**	2.9		50	1.7	2.1	**1.4**	1.8
	75	5.0	5.1	4.8	**4.8**		75	3.0	4.4	**2.5**	3.0
UMich #1 LC	25	1.4	1.5	**1.4**	1.6	KITTI #9	25	1.7	2.1	**1.1**	1.1
	50	**2.5**	2.7	2.6	2.9		50	3.5	4.4	2.0	**2.0**
	75	**5.3**	5.1	5.7	5.6		75	8.1	12.1	3.7	**3.3**
UMich #2 LC	25	2.1	2.2	**2.1**	2.4	RobotCar 05-14 13:46	25	4.5	3.3	2.4	**2.2**
	50	**4.1**	4.7	4.1	4.9		50	9.3	6.4	3.4	**2.8**
	75	**8.2**	10.8	8.5	10.1		75	26.0	20.9	5.5	**3.6**
KITTI #1	25	1.2	1.3	1.0	**0.9**	RobotCar 06-26 08:53	25	3.7	2.5	2.7	**2.3**
	50	2.3	3.0	1.8	**1.7**		50	5.6	3.6	3.9	**2.9**
	75	4.8	8.7	3.2	**3.1**		75	10.1	5.6	6.8	**3.7**

Table 2. Translation error angle (deg) quantiles on outdoor autonomous driving real data