Supplementary materials of Efficient Residue Number System Based Winograd Convolution

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1 Examples of the Winograd Transform over RNS

We provide some example Winograd transform matrices for $F(10 \times 10, 3 \times 3)$, $F(14 \times 14, 3 \times 3)$ and $F(10 \times 10, 7 \times 7)$ and their corresponding transforms over RNS.

1.1 Winograd convolution F(10x10,3x3):

$$y = g \circledast d = A^T \left[[GgG^T] \odot [B^T dB] \right] A$$

$$A^T = \begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
0 & 1 & -1 & 2 & -2 & 3 & -3 & 4 & -4 & 5 & -5 & 0 \\
0 & 1 & 1 & 4 & 4 & 9 & 9 & 16 & 16 & 25 & 25 & 0 \\
0 & 1 & -1 & 8 & -8 & 27 & -27 & 64 & -64 & 125 & -125 & 0 \\
0 & 1 & 1 & 16 & 16 & 81 & 81 & 256 & 256 & 625 & 625 & 0 \\
0 & 1 & -1 & 32 & -32 & 243 & -243 & 1024 & -1024 & 3125 & -3125 & 0 \\
0 & 1 & 1 & 64 & 64 & 729 & 729 & 4096 & 4096 & 15625 & 15625 & 0 \\
0 & 1 & -1 & 128 & -128 & 2187 & -2187 & 16384 & -16384 & 78125 & -78125 & 0 \\
0 & 1 & 1 & 256 & 256 & 6561 & 6561 & 65536 & 65536 & 390625 & 390625 & 0 \\
0 & 1 & -1 & 512 & -512 & 19683 & -19683 & 262144 & -262144 & 1953125 & -1953125 & 1
\end{pmatrix}$$

$$G = \begin{pmatrix}
\frac{1}{14400} & 0 & 0 \\
\frac{1}{17280} & \frac{1}{17280} & \frac{1}{17280} \\
\frac{1}{17280} & \frac{-1}{17280} & \frac{1}{17280} \\
\frac{1}{30240} & \frac{1}{15120} & \frac{1}{7560} \\
\frac{1}{30240} & \frac{-1}{15120} & \frac{1}{7560} \\
\frac{1}{80640} & \frac{1}{26880} & \frac{1}{8960} \\
\frac{1}{80640} & \frac{-1}{26880} & \frac{1}{8960} \\
\frac{1}{362880} & \frac{1}{90720} & \frac{1}{22680} \\
\frac{1}{362880} & \frac{-1}{90720} & \frac{1}{22680} \\
\frac{1}{3628800} & \frac{1}{725760} & \frac{1}{145152} \\
\frac{1}{3628800} & \frac{-1}{725760} & \frac{1}{145152} \\
0 & 0 & 1
\end{pmatrix} = \frac{1}{3628800} \begin{pmatrix}
252 & 0 & 0 \\
210 & 210 & 210 \\
210 & -210 & 210 \\
120 & 240 & 480 \\
120 & -240 & 480 \\
45 & 135 & 405 \\
45 & -135 & 405 \\
10 & 40 & 160 \\
10 & -40 & 160 \\
1 & 5 & 25 \\
1 & -5 & 25 \\
0 & 0 & 3628800
\end{pmatrix}$$

$$B^T = \begin{pmatrix}
14400 & 0 & -21076 & 0 & 7645 & 0 & -1023 & 0 & 55 & 0 & -1 & 0 \\
0 & 14400 & 14400 & -6676 & -6676 & 969 & 969 & -54 & -54 & 1 & 1 & 0 \\
0 & -14400 & 14400 & 6676 & -6676 & -969 & 969 & 54 & -54 & -1 & 1 & 0 \\
0 & -7200 & -3600 & 8738 & 4369 & -1638 & -819 & 102 & 51 & -2 & -1 & 0 \\
0 & 7200 & -3600 & -8738 & 4369 & 1638 & -819 & -102 & 51 & 2 & -1 & 0 \\
0 & 4800 & 1600 & -6492 & -2164 & 1827 & 609 & -138 & -46 & 3 & 1 & 0 \\
0 & -4800 & 1600 & 6492 & -2164 & -1827 & 609 & 138 & -46 & -3 & 1 & 0 \\
0 & -3600 & -900 & 5044 & 1261 & -1596 & -399 & 156 & 39 & -4 & -1 & 0 \\
0 & 3600 & -900 & -5044 & 1261 & 1596 & -399 & -156 & 39 & 4 & -1 & 0 \\
0 & 2880 & 576 & -4100 & -820 & 1365 & 273 & -150 & -30 & 5 & 1 & 0 \\
0 & -2880 & 576 & 4100 & -820 & -1365 & 273 & 150 & -30 & -5 & 1 & 0 \\
0 & -14400 & 0 & 21076 & 0 & -7645 & 0 & 1023 & 0 & -55 & 0 & 1
\end{pmatrix}$$

Winograd convolution with modulus(253), F(10x10,3x3):

$$y_{253} = g \circledast d \pmod{253} = A_{253}^T \Big[[G_{253} g G_{253}^T] \odot [B_{253}^T d B_{253}] \Big] A_{253}$$

$$A_{253}^T = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 2 & -2 & 3 & -3 & 4 & -4 & 5 & -5 & 0 \\ 0 & 1 & 1 & 4 & 4 & 9 & 9 & 16 & 16 & 25 & 25 & 0 \\ 0 & 1 & -1 & 8 & -8 & 27 & -27 & 64 & -64 & 125 & -125 & 0 \\ 0 & 1 & 1 & 16 & 16 & 81 & 81 & 3 & 3 & 119 & 119 & 0 \\ 0 & 1 & -1 & 32 & -32 & -10 & 10 & 12 & -12 & 89 & -89 & 0 \\ 0 & 1 & 1 & 64 & 64 & -30 & -30 & 48 & 48 & -61 & -61 & 0 \\ 0 & 1 & -1 & -125 & 125 & -90 & 90 & -61 & 61 & -52 & 52 & 0 \\ 0 & 1 & 1 & 3 & 3 & -17 & -17 & 9 & 9 & -7 & -7 & 0 \\ 0 & 1 & -1 & 6 & -6 & -51 & 51 & 36 & -36 & -35 & 35 & 1 \end{pmatrix}$$

$$G_{253} = \begin{pmatrix} 12 & 0 & 0 \\ 10 & 10 & 10 \\ 10 & -10 & 10 \\ 78 & -97 & 59 \\ 78 & 97 & 59 \\ -34 & -102 & -53 \\ -34 & 102 & -53 \\ -120 & 26 & 104 \\ -120 & -26 & 104 \\ -12 & -60 & -47 \\ -12 & 60 & -47 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B_{253}^T = \begin{pmatrix} -21 & 0 & -77 & 0 & 55 & 0 & -11 & 0 & 55 & 0 & -1 & 0 \\ 0 & -21 & -21 & -98 & -98 & -43 & -43 & -54 & -54 & 1 & 1 & 0 \\ 0 & 21 & -21 & 98 & -98 & 43 & -43 & 54 & -54 & -1 & 1 & 0 \\ 0 & -116 & -58 & -117 & 68 & -120 & -60 & 102 & 51 & -2 & -1 & 0 \\ 0 & 116 & -58 & 117 & 68 & 120 & -60 & -102 & 51 & 2 & -1 & 0 \\ 0 & -7 & 82 & 86 & 113 & 56 & 103 & 115 & -46 & 3 & 1 & 0 \\ 0 & 7 & 82 & -86 & 113 & -56 & 103 & -115 & -46 & -3 & 1 & 0 \\ 0 & -58 & 112 & -16 & -4 & -78 & 107 & -97 & 39 & -4 & -1 & 0 \\ 0 & 58 & 112 & 16 & -4 & 78 & 107 & 97 & 39 & 4 & -1 & 0 \\ 0 & 97 & 70 & -52 & -61 & 100 & 20 & 103 & -30 & 5 & 1 & 0 \\ 0 & -97 & 70 & 52 & -61 & -100 & 20 & -103 & -30 & -5 & 1 & 0 \\ 0 & 21 & 0 & 77 & 0 & -55 & 0 & 11 & 0 & -55 & 0 & 1 \end{pmatrix}$$

Winograd convolution with modulus(251), F(10x10,3x3):

$$y_{251} = g \circledast d \pmod{251} = A_{251}^T \Big[[G_{251} g G_{251}^T] \odot [B_{251}^T d B_{251}] \Big] A_{251}$$

$$A_{251}^T = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 2 & -2 & 3 & -3 & 4 & -4 & 5 & -5 & 0 \\ 0 & 1 & 1 & 4 & 4 & 9 & 9 & 16 & 16 & 25 & 25 & 0 \\ 0 & 1 & -1 & 8 & -8 & 27 & -27 & 64 & -64 & 125 & -125 & 0 \\ 0 & 1 & 1 & 16 & 16 & 81 & 81 & 5 & 5 & 123 & 123 & 0 \\ 0 & 1 & -1 & 32 & -32 & -8 & 8 & 20 & -20 & 113 & -113 & 0 \\ 0 & 1 & 1 & 64 & 64 & -24 & -24 & 80 & 80 & 63 & 63 & 0 \\ 0 & 1 & -1 & -123 & 123 & -72 & 72 & 69 & -69 & 64 & -64 & 0 \\ 0 & 1 & 1 & 5 & 5 & 35 & 35 & 25 & 25 & 69 & 69 & 0 \\ 0 & 1 & -1 & 10 & -10 & 105 & -105 & 100 & -100 & 94 & -94 & 1 \end{pmatrix}$$

$$G_{251} = \begin{pmatrix} 27 & 0 & 0 \\ -103 & -103 & -103 \\ -103 & 103 & -103 \\ -23 & -46 & -92 \\ -23 & 46 & -92 \\ -40 & -120 & -109 \\ -40 & 120 & -109 \\ 19 & 76 & 53 \\ 19 & -76 & 53 \\ 27 & -116 & -78 \\ 27 & 116 & -78 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B_{251}^T = \begin{pmatrix} 93 & 0 & 8 & 0 & 115 & 0 & -19 & 0 & 55 & 0 & -1 & 0 \\ 0 & 93 & 93 & 101 & 101 & -35 & -35 & -54 & -54 & 1 & 1 & 0 \\ 0 & -93 & 93 & -101 & 101 & 35 & -35 & 54 & -54 & -1 & 1 & 0 \\ 0 & 79 & -86 & -47 & 102 & 119 & -66 & 102 & 51 & -2 & -1 & 0 \\ 0 & -79 & -86 & 47 & 102 & -119 & -66 & -102 & 51 & 2 & -1 & 0 \\ 0 & 31 & 94 & 34 & 95 & 70 & 107 & 113 & -46 & 3 & 1 & 0 \\ 0 & -31 & 94 & -34 & 95 & -70 & 107 & -113 & -46 & -3 & 1 & 0 \\ 0 & -86 & 104 & 24 & 6 & -90 & 103 & -95 & 39 & -4 & -1 & 0 \\ 0 & 86 & 104 & -24 & 6 & 90 & 103 & 95 & 39 & 4 & -1 & 0 \\ 0 & 119 & 74 & -84 & -67 & 110 & 22 & 101 & -30 & 5 & 1 & 0 \\ 0 & -119 & 74 & 84 & -67 & -110 & 22 & -101 & -30 & -5 & 1 & 0 \\ 0 & -93 & 0 & -8 & 0 & -115 & 0 & 19 & 0 & -55 & 0 & 1 \end{pmatrix}$$

Winograd convolution with modulus(247), F(10x10,3x3):

$$y_{247} = g \circledast d \pmod{247} = A_{247}^T \Big[[G_{247} g G_{247}^T] \odot [B_{247}^T d B_{247}] \Big] A_{247}$$

$$A_{247}^T = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 2 & -2 & 3 & -3 & 4 & -4 & 5 & -5 & 0 \\ 0 & 1 & 1 & 4 & 4 & 9 & 9 & 16 & 16 & 25 & 25 & 0 \\ 0 & 1 & -1 & 8 & -8 & 27 & -27 & 64 & -64 & -122 & 122 & 0 \\ 0 & 1 & 1 & 16 & 16 & 81 & 81 & 9 & 9 & -116 & -116 & 0 \\ 0 & 1 & -1 & 32 & -32 & -4 & 4 & 36 & -36 & -86 & 86 & 0 \\ 0 & 1 & 1 & 64 & 64 & -12 & -12 & -103 & -103 & 64 & 64 & 0 \\ 0 & 1 & -1 & -119 & 119 & -36 & 36 & 82 & -82 & 73 & -73 & 0 \\ 0 & 1 & 1 & 9 & 9 & -108 & -108 & 81 & 81 & 118 & 118 & 0 \\ 0 & 1 & -1 & 18 & -18 & -77 & 77 & 77 & -77 & 96 & -96 & 1 \end{pmatrix}$$

$$G_{247} = \begin{pmatrix} -10 & 0 & 0 \\ 74 & 74 & 74 \\ 74 & -74 & 74 \\ 7 & 14 & 28 \\ 7 & -14 & 28 \\ -90 & -23 & -69 \\ -90 & 23 & -69 \\ -20 & -80 & -73 \\ -20 & 80 & -73 \\ -2 & -10 & -50 \\ -2 & 10 & -50 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B_{247}^T = \begin{pmatrix} 74 & 0 & -81 & 0 & -12 & 0 & -35 & 0 & 55 & 0 & -1 & 0 \\ 0 & 74 & 74 & -7 & -7 & -19 & -19 & -54 & -54 & 1 & 1 & 0 \\ 0 & -74 & 74 & 7 & -7 & 19 & -19 & 54 & -54 & -1 & 1 & 0 \\ 0 & -37 & 105 & 93 & -77 & 91 & -78 & 102 & 51 & -2 & -1 & 0 \\ 0 & 37 & 105 & -93 & -77 & -91 & -78 & -102 & 51 & 2 & -1 & 0 \\ 0 & 107 & 118 & -70 & 59 & 98 & 115 & 109 & -46 & 3 & 1 & 0 \\ 0 & -107 & 118 & 70 & 59 & -98 & 115 & -109 & -46 & -3 & 1 & 0 \\ 0 & 105 & 88 & 104 & 26 & -114 & 95 & -91 & 39 & -4 & -1 & 0 \\ 0 & -105 & 88 & -104 & 26 & 114 & 95 & 91 & 39 & 4 & -1 & 0 \\ 0 & -84 & 82 & 99 & -79 & -117 & 26 & 97 & -30 & 5 & 1 & 0 \\ 0 & 84 & 82 & -99 & -79 & 117 & 26 & -97 & -30 & -5 & 1 & 0 \\ 0 & -74 & 0 & 81 & 0 & 12 & 0 & 35 & 0 & -55 & 0 & 1 \end{pmatrix}$$

1.2 Winograd convolution F(10x10,3x3):

$$y = g \circledast d = A^T \Big[[GgG^T] \odot [B^T dB] \Big] A$$

$$A^T = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 2 & -2 & 3 & -3 & 4 & -4 & 5 & -5 & 0 \\ 0 & 1 & 1 & 4 & 4 & 9 & 9 & 16 & 16 & 25 & 25 & 0 \\ 0 & 1 & -1 & 8 & -8 & 27 & -27 & 64 & -64 & 125 & -125 & 0 \\ 0 & 1 & 1 & 16 & 16 & 81 & 81 & 256 & 256 & 625 & 625 & 0 \\ 0 & 1 & -1 & 32 & -32 & 243 & -243 & 1024 & -1024 & 3125 & -3125 & 0 \\ 0 & 1 & 1 & 64 & 64 & 729 & 729 & 4096 & 4096 & 15625 & 15625 & 0 \\ 0 & 1 & -1 & 128 & -128 & 2187 & -2187 & 16384 & -16384 & 78125 & -78125 & 0 \\ 0 & 1 & 1 & 256 & 256 & 6561 & 6561 & 65536 & 65536 & 390625 & 390625 & 0 \\ 0 & 1 & -1 & 512 & -512 & 19683 & -19683 & 262144 & -262144 & 1953125 & -1953125 & 1 \end{pmatrix}$$

$$G = \begin{pmatrix} \frac{1}{14400} & 0 & 0 \\ \frac{1}{17280} & \frac{1}{17280} & \frac{1}{17280} \\ \frac{1}{17280} & \frac{-1}{17280} & \frac{1}{17280} \\ \frac{1}{30240} & \frac{1}{15120} & \frac{1}{7560} \\ \frac{1}{30240} & \frac{-1}{15120} & \frac{1}{7560} \\ \frac{1}{80640} & \frac{1}{26880} & \frac{1}{8960} \\ \frac{1}{80640} & \frac{-1}{26880} & \frac{1}{8960} \\ \frac{1}{362880} & \frac{1}{90720} & \frac{1}{22680} \\ \frac{1}{362880} & \frac{-1}{90720} & \frac{1}{22680} \\ \frac{1}{3628800} & \frac{1}{725760} & \frac{1}{145152} \\ \frac{1}{3628800} & \frac{-1}{725760} & \frac{1}{145152} \\ 0 & 0 & 1 \end{pmatrix} = \frac{1}{3628800} \begin{pmatrix} 252 & 0 & 0 \\ 210 & 210 & 210 \\ 210 & -210 & 210 \\ 120 & 240 & 480 \\ 120 & -240 & 480 \\ 45 & 135 & 405 \\ 45 & -135 & 405 \\ 10 & 40 & 160 \\ 10 & -40 & 160 \\ 1 & 5 & 25 \\ 1 & -5 & 25 \\ 0 & 0 & 3628800 \end{pmatrix}$$

$$B^T = \begin{pmatrix} 14400 & 0 & -21076 & 0 & 7645 & 0 & -1023 & 0 & 55 & 0 & -1 & 0 \\ 0 & 14400 & 14400 & -6676 & -6676 & 969 & 969 & -54 & -54 & 1 & 1 & 0 \\ 0 & -14400 & 14400 & 6676 & -6676 & -969 & 969 & 54 & -54 & -1 & 1 & 0 \\ 0 & -7200 & -3600 & 8738 & 4369 & -1638 & -819 & 102 & 51 & -2 & -1 & 0 \\ 0 & 7200 & -3600 & -8738 & 4369 & 1638 & -819 & -102 & 51 & 2 & -1 & 0 \\ 0 & 4800 & 1600 & -6492 & -2164 & 1827 & 609 & -138 & -46 & 3 & 1 & 0 \\ 0 & -4800 & 1600 & 6492 & -2164 & -1827 & 609 & 138 & -46 & -3 & 1 & 0 \\ 0 & -3600 & -900 & 5044 & 1261 & -1596 & -399 & 156 & 39 & -4 & -1 & 0 \\ 0 & 3600 & -900 & -5044 & 1261 & 1596 & -399 & -156 & 39 & 4 & -1 & 0 \\ 0 & 2880 & 576 & -4100 & -820 & 1365 & 273 & -150 & -30 & 5 & 1 & 0 \\ 0 & -2880 & 576 & 4100 & -820 & -1365 & 273 & 150 & -30 & -5 & 1 & 0 \\ 0 & -14400 & 0 & 21076 & 0 & -7645 & 0 & 1023 & 0 & -55 & 0 & 1 \end{pmatrix}$$

Winograd convolution with modulus(4001), F(10x10,3x3):

$$y_{4001} = g \circledast d \pmod{4001} = A_{4001}^T \left[[G_{4001} g G_{4001}^T] \odot [B_{4001}^T d B_{4001}] \right] A_{4001}$$

$$A_{4001}^T = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 2 & -2 & 3 & -3 & 4 & -4 & 5 & -5 & 0 \\ 0 & 1 & 1 & 4 & 4 & 9 & 9 & 16 & 16 & 25 & 25 & 0 \\ 0 & 1 & -1 & 8 & -8 & 27 & -27 & 64 & -64 & 125 & -125 & 0 \\ 0 & 1 & 1 & 16 & 16 & 81 & 81 & 256 & 256 & 625 & 625 & 0 \\ 0 & 1 & -1 & 32 & -32 & 243 & -243 & 1024 & -1024 & -876 & 876 & 0 \\ 0 & 1 & 1 & 64 & 64 & 729 & 729 & 95 & 95 & -379 & -379 & 0 \\ 0 & 1 & -1 & 128 & -128 & -1814 & 1814 & 380 & -380 & -1895 & 1895 & 0 \\ 0 & 1 & 1 & 256 & 256 & -1441 & -1441 & 1520 & 1520 & -1473 & -1473 & 0 \\ 0 & 1 & -1 & 512 & -512 & -322 & 322 & -1922 & 1922 & 637 & -637 & 1 \end{pmatrix}$$

$$G_{4001} = \begin{pmatrix} 222 & 0 & 0 \\ 185 & 185 & 185 \\ 185 & -185 & 185 \\ -1609 & 783 & 1566 \\ -1609 & -783 & 1566 \\ 897 & -1310 & 71 \\ 897 & 1310 & 71 \\ 1533 & -1870 & 522 \\ 1533 & 1870 & 522 \\ -1047 & -1234 & 1832 \\ -1047 & 1234 & 1832 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B_{4001}^T = \begin{pmatrix} -1604 & 0 & -1071 & 0 & -357 & 0 & -1023 & 0 & 55 & 0 & -1 & 0 \\ 0 & -1604 & -1604 & 1326 & 1326 & 969 & 969 & -54 & -54 & 1 & 1 & 0 \\ 0 & 1604 & -1604 & -1326 & 1326 & -969 & 969 & 54 & -54 & -1 & 1 & 0 \\ 0 & 802 & 401 & 736 & 368 & -1638 & -819 & 102 & 51 & -2 & -1 & 0 \\ 0 & -802 & 401 & -736 & 368 & 1638 & -819 & -102 & 51 & 2 & -1 & 0 \\ 0 & 799 & 1600 & 1510 & 1837 & 1827 & 609 & -138 & -46 & 3 & 1 & 0 \\ 0 & -799 & 1600 & -1510 & 1837 & -1827 & 609 & 138 & -46 & -3 & 1 & 0 \\ 0 & 401 & -900 & 1043 & 1261 & -1596 & -399 & 156 & 39 & -4 & -1 & 0 \\ 0 & -401 & -900 & -1043 & 1261 & 1596 & -399 & -156 & 39 & 4 & -1 & 0 \\ 0 & -1121 & 576 & -99 & -820 & 1365 & 273 & -150 & -30 & 5 & 1 & 0 \\ 0 & 1121 & 576 & 99 & -820 & -1365 & 273 & 150 & -30 & -5 & 1 & 0 \\ 0 & 1604 & 0 & 1071 & 0 & 357 & 0 & 1023 & 0 & -55 & 0 & 1 \end{pmatrix}$$

Winograd convolution with modulus(4331), F(10x10,3x3):

$$y_{4331} = g \circledast d \pmod{4331} = A_{4331}^T \Big[[G_{4331} g G_{4331}^T] \odot [B_{4331}^T d B_{4331}] \Big] A_{4331}$$

$$A_{4331}^T = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 2 & -2 & 3 & -3 & 4 & -4 & 5 & -5 & 0 \\ 0 & 1 & 1 & 4 & 4 & 9 & 9 & 16 & 16 & 25 & 25 & 0 \\ 0 & 1 & -1 & 8 & -8 & 27 & -27 & 64 & -64 & 125 & -125 & 0 \\ 0 & 1 & 1 & 16 & 16 & 81 & 81 & 256 & 256 & 625 & 625 & 0 \\ 0 & 1 & -1 & 32 & -32 & 243 & -243 & 1024 & -1024 & -1206 & 1206 & 0 \\ 0 & 1 & 1 & 64 & 64 & 729 & 729 & -235 & -235 & -1699 & -1699 & 0 \\ 0 & 1 & -1 & 128 & -128 & -2144 & 2144 & -940 & 940 & 167 & -167 & 0 \\ 0 & 1 & 1 & 256 & 256 & -2101 & -2101 & 571 & 571 & 835 & 835 & 0 \\ 0 & 1 & -1 & 512 & -512 & -1972 & 1972 & -2047 & 2047 & -156 & 156 & 1 \end{pmatrix}$$

$$G_{4331} = \begin{pmatrix} 1693 & 0 & 0 \\ 689 & 689 & 689 \\ 689 & -689 & 689 \\ -225 & -450 & -900 \\ -225 & 450 & -900 \\ 457 & 1371 & -218 \\ 457 & -1371 & -218 \\ 1064 & -75 & -300 \\ 1064 & 75 & -300 \\ -1626 & 532 & -1671 \\ -1626 & -532 & -1671 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B_{4331}^T = \begin{pmatrix} 1407 & 0 & 579 & 0 & -1017 & 0 & -1023 & 0 & 55 & 0 & -1 & 0 \\ 0 & 1407 & 1407 & 1986 & 1986 & 969 & 969 & -54 & -54 & 1 & 1 & 0 \\ 0 & -1407 & 1407 & -1986 & 1986 & -969 & 969 & 54 & -54 & -1 & 1 & 0 \\ 0 & 1462 & 731 & 76 & 38 & -1638 & -819 & 102 & 51 & -2 & -1 & 0 \\ 0 & -1462 & 731 & -76 & 38 & 1638 & -819 & -102 & 51 & 2 & -1 & 0 \\ 0 & 469 & 1600 & -2161 & -2164 & 1827 & 609 & -138 & -46 & 3 & 1 & 0 \\ 0 & -469 & 1600 & 2161 & -2164 & -1827 & 609 & 138 & -46 & -3 & 1 & 0 \\ 0 & 731 & -900 & 713 & 1261 & -1596 & -399 & 156 & 39 & -4 & -1 & 0 \\ 0 & -731 & -900 & -713 & 1261 & 1596 & -399 & -156 & 39 & 4 & -1 & 0 \\ 0 & -1451 & 576 & 231 & -820 & 1365 & 273 & -150 & -30 & 5 & 1 & 0 \\ 0 & 1451 & 576 & -231 & -820 & -1365 & 273 & 150 & -30 & -5 & 1 & 0 \\ 0 & -1407 & 0 & -579 & 0 & 1017 & 0 & 1023 & 0 & -55 & 0 & 1 \end{pmatrix}$$

1.3 Winograd convolution F(14x14,3x3):

$y = g \circledast d = A^T \Big[[GgG^T] \odot [B^T dB] \Big] A$

$$A^T = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 2 & -2 & 3 & -3 & 4 & -4 & 5 & -5 & 6 & -6 & 7 & -7 & 0 \\ 0 & 1 & 1 & 4 & 4 & 9 & 9 & 16 & 16 & 25 & 25 & 36 & 36 & 49 & 49 & 0 \\ 0 & 1 & -1 & 8 & -8 & 27 & -27 & 64 & -64 & 125 & -125 & 216 & -216 & 343 & -343 & 0 \\ 0 & 1 & 1 & 16 & 16 & 81 & 81 & 256 & 256 & 625 & 625 & 1296 & 1296 & 2401 & 2401 & 0 \\ 0 & 1 & -1 & 32 & -32 & 243 & -243 & 1024 & -1024 & 3125 & -3125 & 7776 & -7776 & 16807 & -16807 & 0 \\ 0 & 1 & 1 & 64 & 64 & 729 & 729 & 4096 & 4096 & 15625 & 15625 & 46656 & 46656 & 117649 & 117649 & 0 \\ 0 & 1 & -1 & 128 & -128 & 2187 & -2187 & 16384 & -16384 & 78125 & -78125 & 279936 & -279936 & 823543 & -823543 & 0 \\ 0 & 1 & 1 & 256 & 256 & 6561 & 6561 & 65536 & 65536 & 390625 & 390625 & 1679616 & 1679616 & 5764801 & 5764801 & 0 \\ 0 & 1 & -1 & 512 & -512 & 19683 & -19683 & 262144 & -262144 & 1953125 & -1953125 & 10077696 & -10077696 & 40353607 & -40353607 & 0 \\ 0 & 1 & 1 & 1024 & 1024 & 59049 & 59049 & 1048576 & 1048576 & 9765625 & 9765625 & 60466176 & 60466176 & 282475249 & 282475249 & 0 \\ 0 & 1 & -1 & 2048 & -2048 & 177147 & -177147 & 4194304 & -4194304 & 48828125 & -48828125 & 362797056 & -362797056 & 1977326743 & -1977326743 & 0 \\ 0 & 1 & 1 & 4096 & 4096 & 531441 & 531441 & 16777216 & 16777216 & 244140625 & 244140625 & 2176782336 & 2176782336 & 13841287201 & 13841287201 & 0 \\ 0 & 1 & -1 & 8192 & -8192 & 1594323 & -1594323 & 67108864 & -67108864 & 1220703125 & -1220703125 & 13060694016 & -13060694016 & 96889010407 & -96889010407 & 1 \end{pmatrix}$$

$$G = \begin{pmatrix} \frac{1}{25401600} & 0 & 0 \\ \frac{1}{29030400} & \frac{1}{29030400} & \frac{1}{29030400} \\ \frac{1}{29030400} & \frac{-1}{29030400} & \frac{1}{29030400} \\ \frac{1}{43545600} & \frac{1}{21772800} & \frac{1}{10886400} \\ \frac{1}{43545600} & \frac{-1}{21772800} & \frac{1}{10886400} \\ \frac{1}{87091200} & \frac{1}{29030400} & \frac{1}{9676800} \\ \frac{1}{87091200} & \frac{-1}{29030400} & \frac{1}{9676800} \\ \frac{1}{239500800} & \frac{1}{59875200} & \frac{1}{14968800} \\ \frac{1}{239500800} & \frac{-1}{59875200} & \frac{1}{14968800} \\ \frac{1}{958003200} & \frac{1}{191600640} & \frac{1}{38320128} \\ \frac{1}{958003200} & \frac{-1}{191600640} & \frac{1}{38320128} \\ \frac{1}{6227020800} & \frac{1}{1037836800} & \frac{1}{172972800} \\ \frac{1}{6227020800} & \frac{-1}{1037836800} & \frac{1}{172972800} \\ \frac{1}{87178291200} & \frac{1}{12454041600} & \frac{1}{1779148800} \\ \frac{1}{87178291200} & \frac{-1}{12454041600} & \frac{1}{1779148800} \\ 0 & 0 & 1 \end{pmatrix} = \frac{1}{87178291200} \begin{pmatrix} 3432 & 0 & 0 \\ 3003 & 3003 & 3003 \\ 3003 & -3003 & 3003 \\ 2002 & 4004 & 8008 \\ 2002 & -4004 & 8008 \\ 1001 & 3003 & 9009 \\ 1001 & -3003 & 9009 \\ 364 & 1456 & 5824 \\ 364 & -1456 & 5824 \\ 91 & 455 & 2275 \\ 91 & -455 & 2275 \\ 14 & 84 & 504 \\ 14 & -84 & 504 \\ 1 & 7 & 49 \\ 1 & -7 & 49 \\ 0 & 0 & 87178291200 \end{pmatrix}$$

$$B^T = \begin{pmatrix} 25401600 & 0 & -38402064 & 0 & 15291640 & 0 & -2475473 & 0 & 191620 & 0 & -7462 & 0 & 140 & 0 & -1 & 0 \\ 0 & 25401600 & 25401600 & -13000464 & -13000464 & 2291176 & 2291176 & -184297 & -184297 & 7323 & 7323 & -139 & -139 & 1 & 1 & 0 \\ 0 & -25401600 & 25401600 & 13000464 & -13000464 & -2291176 & 2291176 & 184297 & -184297 & -7323 & 7323 & 139 & -139 & -1 & 1 & 0 \\ 0 & -12700800 & -6350400 & 16025832 & 8012916 & -3639362 & -1819681 & 327896 & 163948 & -13836 & -6918 & 272 & 136 & -2 & -1 & 0 \\ 0 & 12700800 & -6350400 & -16025832 & 8012916 & 3639362 & -1819681 & -327896 & 163948 & 13836 & -6918 & -272 & 136 & 2 & -1 & 0 \\ 0 & 8467200 & 2822400 & -11859888 & -3953296 & 3779448 & 1259816 & -405219 & -135073 & 18849 & 6283 & -393 & -131 & 3 & 1 & 0 \\ 0 & -8467200 & 2822400 & 11859888 & -3953296 & -3779448 & 1259816 & 405219 & -135073 & -18849 & 6283 & 393 & -131 & -3 & 1 & 0 \\ 0 & -6350400 & -1587600 & 9203616 & 2300904 & -3247684 & -811921 & 415888 & 103972 & -21912 & -5478 & 496 & 124 & -4 & -1 & 0 \\ 0 & 6350400 & -1587600 & -9203616 & 2300904 & 3247684 & -811921 & -415888 & 103972 & 21912 & -5478 & -496 & 124 & 4 & -1 & 0 \\ 0 & 5080320 & 1016064 & -7477200 & -1495440 & 2759240 & 551848 & -384725 & -76945 & 22935 & 4587 & -575 & -115 & 5 & 1 & 0 \\ 0 & -5080320 & 1016064 & 7477200 & -1495440 & -2759240 & 551848 & 384725 & -76945 & -22935 & 4587 & 575 & -115 & -5 & 1 & 0 \\ 0 & -4233600 & -705600 & 6282744 & 1047124 & -2374086 & -395681 & 346632 & 57772 & -22308 & -3718 & 624 & 104 & -6 & -1 & 0 \\ 0 & 4233600 & -705600 & -6282744 & 1047124 & 2374086 & -395681 & -346632 & 57772 & 22308 & -3718 & -624 & 104 & 6 & -1 & 0 \\ 0 & 3628800 & 518400 & -5411952 & -773136 & 2074072 & 296296 & -311311 & -44473 & 21021 & 3003 & -637 & -91 & 7 & 1 & 0 \\ 0 & -3628800 & 518400 & 5411952 & -773136 & -2074072 & 296296 & 311311 & -44473 & -21021 & 3003 & 637 & -91 & -7 & 1 & 0 \\ 0 & -25401600 & 0 & 38402064 & 0 & -15291640 & 0 & 2475473 & 0 & -191620 & 0 & 7462 & 0 & -140 & 0 & 1 \end{pmatrix}$$

Winograd convolution with modulus(251), F(14x14,3x3):

$$y_{251} = g \circledast d \pmod{251} = A_{251}^T \Big[[G_{251} g G_{251}^T] \odot [B_{251}^T d B_{251}] \Big] A_{251}$$

$$A_{251}^T = \begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
0 & 1 & -1 & 2 & -2 & 3 & -3 & 4 & -4 & 5 & -5 & 6 & -6 & 7 & -7 & 0 \\
0 & 1 & 1 & 4 & 4 & 9 & 9 & 16 & 16 & 25 & 25 & 36 & 36 & 49 & 49 & 0 \\
0 & 1 & -1 & 8 & -8 & 27 & -27 & 64 & -64 & 125 & -125 & -35 & 35 & 92 & -92 & 0 \\
0 & 1 & 1 & 16 & 16 & 81 & 81 & 5 & 5 & 123 & 123 & 41 & 41 & -109 & -109 & 0 \\
0 & 1 & -1 & 32 & -32 & -8 & 8 & 20 & -20 & 113 & -113 & -5 & 5 & -10 & 10 & 0 \\
0 & 1 & 1 & 64 & 64 & -24 & -24 & 80 & 80 & 63 & 63 & -30 & -30 & -70 & -70 & 0 \\
0 & 1 & -1 & -123 & 123 & -72 & 72 & 69 & -69 & 64 & -64 & 71 & -71 & 12 & -12 & 0 \\
0 & 1 & 1 & 5 & 5 & 35 & 35 & 25 & 25 & 69 & 69 & -76 & -76 & 84 & 84 & 0 \\
0 & 1 & -1 & 10 & -10 & 105 & -105 & 100 & -100 & 94 & -94 & 46 & -46 & 86 & -86 & 0 \\
0 & 1 & 1 & 20 & 20 & 64 & 64 & -102 & -102 & -32 & -32 & 25 & 25 & 100 & 100 & 0 \\
0 & 1 & -1 & 40 & -40 & -59 & 59 & 94 & -94 & 91 & -91 & -101 & 101 & -53 & 53 & 0 \\
0 & 1 & 1 & 80 & 80 & 74 & 74 & 125 & 125 & -47 & -47 & -104 & -104 & -120 & -120 & 0 \\
0 & 1 & -1 & -91 & 91 & -29 & 29 & -2 & 2 & 16 & -16 & -122 & 122 & -87 & 87 & 1
\end{pmatrix}$$

$$G_{251} = \begin{pmatrix}
-32 & 0 & 0 \\
-28 & -28 & -28 \\
-28 & 28 & -28 \\
65 & -121 & 9 \\
65 & 121 & 9 \\
-93 & -28 & -84 \\
-93 & 28 & -84 \\
-11 & -44 & 75 \\
-11 & 44 & 75 \\
60 & 49 & -6 \\
60 & -49 & -6 \\
-68 & 94 & 62 \\
-68 & -94 & 62 \\
31 & -34 & 13 \\
31 & 34 & 13 \\
0 & 0 & 1
\end{pmatrix}$$

$$B_{251}^T = \begin{pmatrix}
-102 & 0 & -68 & 0 & -33 & 0 & -111 & 0 & 107 & 0 & 68 & 0 & -111 & 0 & -1 & 0 \\
0 & -102 & -102 & 81 & 81 & 48 & 48 & -63 & -63 & 44 & 44 & 112 & 112 & 1 & 1 & 0 \\
0 & 102 & -102 & -81 & 81 & -48 & 48 & 63 & -63 & -44 & 44 & -112 & 112 & -1 & 1 & 0 \\
0 & 51 & -100 & -16 & -8 & -113 & 69 & 90 & 45 & -31 & 110 & 21 & -115 & -2 & -1 & 0 \\
0 & -51 & -100 & 16 & -8 & 113 & 69 & -90 & 45 & 31 & 110 & -21 & -115 & 2 & -1 & 0 \\
0 & -34 & -95 & 113 & -46 & -110 & 47 & -105 & -35 & 24 & 8 & 109 & 120 & 3 & 1 & 0 \\
0 & 34 & -95 & -113 & -46 & 110 & 47 & 105 & -35 & -24 & 8 & -109 & 120 & -3 & 1 & 0 \\
0 & -100 & -25 & -52 & -13 & 5 & 64 & -19 & 58 & -75 & 44 & -6 & 124 & -4 & -1 & 0 \\
0 & 100 & -25 & 52 & -13 & -5 & 64 & 19 & 58 & 75 & 44 & 6 & 124 & 4 & -1 & 0 \\
0 & 80 & 16 & 90 & 18 & -3 & -101 & 58 & 112 & 94 & 69 & -73 & -115 & 5 & 1 & 0 \\
0 & -80 & 16 & -90 & 18 & 3 & -101 & -58 & 112 & -94 & 69 & 73 & -115 & -5 & 1 & 0 \\
0 & 17 & -39 & -37 & -48 & 123 & -105 & 1 & 42 & 31 & 47 & 122 & 104 & -6 & -1 & 0 \\
0 & -17 & -39 & 37 & -48 & -123 & -105 & -1 & 42 & -31 & 47 & -122 & 104 & 6 & -1 & 0 \\
0 & 93 & 85 & 110 & -56 & 59 & 116 & -71 & -46 & -63 & -9 & 116 & -91 & 7 & 1 & 0 \\
0 & -93 & 85 & -110 & -56 & -59 & 116 & 71 & -46 & 63 & -9 & -116 & -91 & -7 & 1 & 0 \\
0 & 102 & 0 & 68 & 0 & 33 & 0 & 111 & 0 & -107 & 0 & -68 & 0 & 111 & 0 & 1
\end{pmatrix}$$

Winograd convolution with modulus(241), F(14x14,3x3):

$$y_{241} = g \circledast d \pmod{241} = A_{241}^T \Big[[G_{241} g G_{241}^T] \odot [B_{241}^T d B_{241}] \Big] A_{241}$$

$$A_{241}^T = \begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
0 & 1 & -1 & 2 & -2 & 3 & -3 & 4 & -4 & 5 & -5 & 6 & -6 & 7 & -7 & 0 \\
0 & 1 & 1 & 4 & 4 & 9 & 9 & 16 & 16 & 25 & 25 & 36 & 36 & 49 & 49 & 0 \\
0 & 1 & -1 & 8 & -8 & 27 & -27 & 64 & -64 & -116 & 116 & -25 & 25 & 102 & -102 & 0 \\
0 & 1 & 1 & 16 & 16 & 81 & 81 & 15 & 15 & -98 & -98 & 91 & 91 & -9 & -9 & 0 \\
0 & 1 & -1 & 32 & -32 & 2 & -2 & 60 & -60 & -8 & 8 & 64 & -64 & -63 & 63 & 0 \\
0 & 1 & 1 & 64 & 64 & 6 & 6 & -1 & -1 & -40 & -40 & -98 & -98 & 41 & 41 & 0 \\
0 & 1 & -1 & -113 & 113 & 18 & -18 & -4 & 4 & 41 & -41 & -106 & 106 & 46 & -46 & 0 \\
0 & 1 & 1 & 15 & 15 & 54 & 54 & -16 & -16 & -36 & -36 & 87 & 87 & 81 & 81 & 0 \\
0 & 1 & -1 & 30 & -30 & -79 & 79 & -64 & 64 & 61 & -61 & 40 & -40 & 85 & -85 & 0 \\
0 & 1 & 1 & 60 & 60 & 4 & 4 & -15 & -15 & 64 & 64 & -1 & -1 & 113 & 113 & 0 \\
0 & 1 & -1 & 120 & -120 & 12 & -12 & -60 & 60 & 79 & -79 & -6 & 6 & 68 & -68 & 0 \\
0 & 1 & 1 & -1 & -1 & 36 & 36 & 1 & 1 & -87 & -87 & -36 & -36 & -6 & -6 & 0 \\
0 & 1 & -1 & -2 & 2 & 108 & -108 & 4 & -4 & 47 & -47 & 25 & -25 & -42 & 42 & 1
\end{pmatrix}$$

$$G_{241} = \begin{pmatrix}
47 & 0 & 0 \\
11 & 11 & 11 \\
11 & -11 & 11 \\
-73 & 95 & -51 \\
-73 & -95 & -51 \\
84 & 11 & 33 \\
84 & -11 & 33 \\
-79 & -75 & -59 \\
-79 & 75 & -59 \\
-80 & 82 & -72 \\
-80 & -82 & -72 \\
-105 & 93 & 76 \\
-105 & -93 & 76 \\
113 & 68 & -6 \\
113 & -68 & -6 \\
0 & 0 & 1
\end{pmatrix}$$

$$B_{241}^T = \begin{pmatrix}
-41 & 0 & 81 & 0 & -51 & 0 & 79 & 0 & 25 & 0 & 9 & 0 & -101 & 0 & -1 & 0 \\
0 & -41 & -41 & 40 & 40 & -11 & -11 & 68 & 68 & 93 & 93 & 102 & 102 & 1 & 1 & 0 \\
0 & 41 & -41 & -40 & 40 & 11 & -11 & -68 & 68 & -93 & 93 & -102 & 102 & -1 & 1 & 0 \\
0 & -100 & -50 & 55 & -93 & -21 & 110 & -105 & 68 & -99 & 71 & 31 & -105 & -2 & -1 & 0 \\
0 & 100 & -50 & -55 & -93 & 21 & 110 & 105 & 68 & 99 & 71 & -31 & -105 & 2 & -1 & 0 \\
0 & -94 & 49 & -37 & 68 & 86 & 109 & -98 & -113 & 51 & 17 & 89 & 110 & 3 & 1 & 0 \\
0 & 94 & 49 & 37 & 68 & -86 & 109 & 98 & -113 & -51 & 17 & -89 & 110 & -3 & 1 & 0 \\
0 & -50 & 108 & 67 & 77 & 32 & 8 & -78 & 101 & 19 & 65 & 14 & -117 & -4 & -1 & 0 \\
0 & 50 & 108 & -67 & 77 & -32 & 8 & 78 & 101 & -19 & 65 & -14 & -117 & 4 & -1 & 0 \\
0 & 40 & 8 & 66 & -35 & 31 & -42 & -89 & -66 & 40 & 8 & -93 & -115 & 5 & 1 & 0 \\
0 & -40 & 8 & -66 & -35 & -31 & -42 & 89 & -66 & -40 & 8 & 93 & -115 & -5 & 1 & 0 \\
0 & 47 & 48 & 115 & -21 & 5 & 41 & 74 & -68 & 105 & -103 & -99 & 104 & -6 & -1 & 0 \\
0 & -47 & 48 & -115 & -21 & -5 & 41 & -74 & -68 & -105 & -103 & 99 & 104 & 6 & -1 & 0 \\
0 & 63 & 9 & -56 & -8 & 26 & 107 & 61 & 112 & 54 & 111 & 86 & -91 & 7 & 1 & 0 \\
0 & -63 & 9 & 56 & -8 & -26 & 107 & -61 & 112 & -54 & 111 & -86 & -91 & -7 & 1 & 0 \\
0 & 41 & 0 & -81 & 0 & 51 & 0 & -79 & 0 & -25 & 0 & -9 & 0 & 101 & 0 & 1
\end{pmatrix}$$

Winograd convolution with modulus(239), F(14x14,3x3):

$$y_{239} = g \circledast d \pmod{239} = A_{239}^T \Big[[G_{239} g G_{239}^T] \odot [B_{239}^T d B_{239}] \Big] A_{239}$$

$$A_{239}^T = \begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
0 & 1 & -1 & 2 & -2 & 3 & -3 & 4 & -4 & 5 & -5 & 6 & -6 & 7 & -7 & 0 \\
0 & 1 & 1 & 4 & 4 & 9 & 9 & 16 & 16 & 25 & 25 & 36 & 36 & 49 & 49 & 0 \\
0 & 1 & -1 & 8 & -8 & 27 & -27 & 64 & -64 & -114 & 114 & -23 & 23 & 104 & -104 & 0 \\
0 & 1 & 1 & 16 & 16 & 81 & 81 & 17 & 17 & -92 & -92 & 101 & 101 & 11 & 11 & 0 \\
0 & 1 & -1 & 32 & -32 & 4 & -4 & 68 & -68 & 18 & -18 & -111 & 111 & 77 & -77 & 0 \\
0 & 1 & 1 & 64 & 64 & 12 & 12 & 33 & 33 & 90 & 90 & 51 & 51 & 61 & 61 & 0 \\
0 & 1 & -1 & -111 & 111 & 36 & -36 & -107 & 107 & -28 & 28 & 67 & -67 & -51 & 51 & 0 \\
0 & 1 & 1 & 17 & 17 & 108 & 108 & 50 & 50 & 99 & 99 & -76 & -76 & -118 & -118 & 0 \\
0 & 1 & -1 & 34 & -34 & 85 & -85 & -39 & 39 & 17 & -17 & 22 & -22 & -109 & 109 & 0 \\
0 & 1 & 1 & 68 & 68 & 16 & 16 & 83 & 83 & 85 & 85 & -107 & -107 & -46 & -46 & 0 \\
0 & 1 & -1 & -103 & 103 & 48 & -48 & 93 & -93 & -53 & 53 & 75 & -75 & -83 & 83 & 0 \\
0 & 1 & 1 & 33 & 33 & -95 & -95 & -106 & -106 & -26 & -26 & -28 & -28 & -103 & -103 & 0 \\
0 & 1 & -1 & 66 & -66 & -46 & 46 & 54 & -54 & 109 & -109 & 71 & -71 & -4 & 4 & 1
\end{pmatrix}$$

$$G_{239} = \begin{pmatrix}
-84 & 0 & 0 \\
46 & 46 & 46 \\
46 & -46 & 46 \\
-49 & -98 & 43 \\
-49 & 98 & 43 \\
95 & 46 & -101 \\
95 & -46 & -101 \\
78 & 73 & 53 \\
78 & -73 & 53 \\
-100 & -22 & -110 \\
-100 & 22 & -110 \\
3 & 18 & 108 \\
3 & -18 & 108 \\
-51 & -118 & -109 \\
-51 & 118 & -109 \\
0 & 0 & 1
\end{pmatrix}$$

$$B_{239}^T = \begin{pmatrix}
-37 & 0 & -22 & 0 & -58 & 0 & 89 & 0 & -58 & 0 & -53 & 0 & -99 & 0 & -1 & 0 \\
0 & -37 & -37 & -59 & -59 & -117 & -117 & -28 & -28 & -86 & -86 & 100 & 100 & 1 & 1 & 0 \\
0 & 37 & -37 & 59 & -59 & 117 & -117 & 28 & -28 & 86 & -86 & -100 & 100 & -1 & 1 & 0 \\
0 & -101 & 69 & -74 & -37 & -109 & 65 & -12 & -6 & 26 & 13 & 33 & -103 & -2 & -1 & 0 \\
0 & 101 & 69 & 74 & -37 & 109 & 65 & 12 & -6 & -26 & 13 & -33 & -103 & 2 & -1 & 0 \\
0 & -92 & 49 & 9 & 3 & -98 & 47 & -114 & -38 & -32 & 69 & 85 & 108 & 3 & 1 & 0 \\
0 & 92 & 49 & -9 & 3 & 98 & 47 & 114 & -38 & 32 & 69 & -85 & 108 & -3 & 1 & 0 \\
0 & 69 & 77 & -35 & 51 & 87 & -38 & 28 & 7 & 76 & 19 & 18 & -115 & -4 & -1 & 0 \\
0 & -69 & 77 & 35 & 51 & -87 & -38 & -28 & 7 & -76 & 19 & -18 & -115 & 4 & -1 & 0 \\
0 & -103 & 75 & -85 & -17 & -15 & -3 & 65 & 13 & -9 & 46 & -97 & -115 & 5 & 1 & 0 \\
0 & 103 & 75 & 85 & -17 & 15 & -3 & -65 & 13 & 9 & 46 & 97 & -115 & -5 & 1 & 0 \\
0 & 46 & -72 & -88 & 65 & -99 & 103 & 82 & -66 & -81 & 106 & -93 & 104 & -6 & -1 & 0 \\
0 & -46 & -72 & 88 & 65 & 99 & 103 & -82 & -66 & 81 & 106 & 93 & 104 & 6 & -1 & 0 \\
0 & 63 & 9 & -36 & 29 & 30 & -64 & 106 & -19 & -11 & -104 & 80 & -91 & 7 & 1 & 0 \\
0 & -63 & 9 & 36 & 29 & -30 & -64 & -106 & -19 & 11 & -104 & -80 & -91 & -7 & 1 & 0 \\
0 & 37 & 0 & 22 & 0 & 58 & 0 & -89 & 0 & 58 & 0 & 53 & 0 & 99 & 0 & 1
\end{pmatrix}$$

1.4 Winograd convolution F(14x14,3x3):

$y = g \circledast d = A^T \Big[[GgG^T] \odot [B^T dB] \Big] A$

$$A^T = \begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
0 & 1 & -1 & 2 & -2 & 3 & -3 & 4 & -4 & 5 & -5 & 6 & -6 & 7 & -7 & 0 \\
0 & 1 & 1 & 4 & 4 & 9 & 9 & 16 & 16 & 25 & 25 & 36 & 36 & 49 & 49 & 0 \\
0 & 1 & -1 & 8 & -8 & 27 & -27 & 64 & -64 & 125 & -125 & 216 & -216 & 343 & -343 & 0 \\
0 & 1 & 1 & 16 & 16 & 81 & 81 & 256 & 256 & 625 & 625 & 1296 & 1296 & 2401 & 2401 & 0 \\
0 & 1 & -1 & 32 & -32 & 243 & -243 & 1024 & -1024 & 3125 & -3125 & 7776 & -7776 & 16807 & -16807 & 0 \\
0 & 1 & 1 & 64 & 64 & 729 & 729 & 4096 & 4096 & 15625 & 15625 & 46656 & 46656 & 117649 & 117649 & 0 \\
0 & 1 & -1 & 128 & -128 & 2187 & -2187 & 16384 & -16384 & 78125 & -78125 & 279936 & -279936 & 823543 & -823543 & 0 \\
0 & 1 & 1 & 256 & 256 & 6561 & 6561 & 65536 & 65536 & 390625 & 390625 & 1679616 & 1679616 & 5764801 & 5764801 & 0 \\
0 & 1 & -1 & 512 & -512 & 19683 & -19683 & 262144 & -262144 & 1953125 & -1953125 & 10077696 & -10077696 & 40353607 & -40353607 & 0 \\
0 & 1 & 1 & 1024 & 1024 & 59049 & 59049 & 1048576 & 1048576 & 9765625 & 9765625 & 60466176 & 60466176 & 282475249 & 282475249 & 0 \\
0 & 1 & -1 & 2048 & -2048 & 177147 & -177147 & 4194304 & -4194304 & 48828125 & -48828125 & 362797056 & -362797056 & 1977326743 & -1977326743 & 0 \\
0 & 1 & 1 & 4096 & 4096 & 531441 & 531441 & 16777216 & 16777216 & 244140625 & 244140625 & 2176782336 & 2176782336 & 13841287201 & 13841287201 & 0 \\
0 & 1 & -1 & 8192 & -8192 & 1594323 & -1594323 & 67108864 & -67108864 & 1220703125 & -1220703125 & 13060694016 & -13060694016 & 96889010407 & -96889010407 & 1
\end{pmatrix}$$

$$G = \begin{pmatrix}
\frac{1}{25401600} & 0 & 0 \\
\frac{1}{29030400} & \frac{1}{29030400} & \frac{1}{29030400} \\
\frac{1}{29030400} & \frac{-1}{29030400} & \frac{1}{29030400} \\
\frac{1}{43545600} & \frac{1}{21772800} & \frac{1}{10886400} \\
\frac{1}{43545600} & \frac{-1}{21772800} & \frac{1}{10886400} \\
\frac{1}{87091200} & \frac{1}{29030400} & \frac{1}{9676800} \\
\frac{1}{87091200} & \frac{-1}{29030400} & \frac{1}{9676800} \\
\frac{1}{239500800} & \frac{1}{59875200} & \frac{1}{14968800} \\
\frac{1}{239500800} & \frac{-1}{59875200} & \frac{1}{14968800} \\
\frac{1}{958003200} & \frac{1}{191600640} & \frac{1}{38320128} \\
\frac{1}{958003200} & \frac{-1}{191600640} & \frac{1}{38320128} \\
\frac{1}{6227020800} & \frac{1}{1037836800} & \frac{1}{172972800} \\
\frac{1}{6227020800} & \frac{-1}{1037836800} & \frac{1}{172972800} \\
\frac{1}{87178291200} & \frac{1}{12454041600} & \frac{1}{1779148800} \\
\frac{1}{87178291200} & \frac{-1}{12454041600} & \frac{1}{1779148800} \\
0 & 0 & 1
\end{pmatrix} = \frac{1}{87178291200} \begin{pmatrix}
3432 & 0 & 0 \\
3003 & 3003 & 3003 \\
3003 & -3003 & 3003 \\
2002 & 4004 & 8008 \\
2002 & -4004 & 8008 \\
1001 & 3003 & 9009 \\
1001 & -3003 & 9009 \\
364 & 1456 & 5824 \\
364 & -1456 & 5824 \\
91 & 455 & 2275 \\
91 & -455 & 2275 \\
14 & 84 & 504 \\
14 & -84 & 504 \\
1 & 7 & 49 \\
1 & -7 & 49 \\
0 & 0 & 87178291200
\end{pmatrix}$$

$$B^T = \begin{pmatrix}
25401600 & 0 & -38402064 & 0 & 15291640 & 0 & -2475473 & 0 & 191620 & 0 & -7462 & 0 & 140 & 0 & -1 & 0 \\
0 & 25401600 & 25401600 & -13000464 & -13000464 & 2291176 & 2291176 & -184297 & -184297 & 7323 & 7323 & -139 & -139 & 1 & 1 & 0 \\
0 & -25401600 & 25401600 & 13000464 & -13000464 & -2291176 & 2291176 & 184297 & -184297 & -7323 & 7323 & 139 & -139 & -1 & 1 & 0 \\
0 & -12700800 & -6350400 & 16025832 & 8012916 & -3639362 & -1819681 & 327896 & 163948 & -13836 & -6918 & 272 & 136 & -2 & -1 & 0 \\
0 & 12700800 & -6350400 & -16025832 & 8012916 & 3639362 & -1819681 & -327896 & 163948 & 13836 & -6918 & -272 & 136 & 2 & -1 & 0 \\
0 & 8467200 & 2822400 & -11859888 & -3953296 & 3779448 & 1259816 & -405219 & -135073 & 18849 & 6283 & -393 & -131 & 3 & 1 & 0 \\
0 & -8467200 & 2822400 & 11859888 & -3953296 & -3779448 & 1259816 & 405219 & -135073 & -18849 & 6283 & 393 & -131 & -3 & 1 & 0 \\
0 & -6350400 & -1587600 & 9203616 & 2300904 & -3247684 & -811921 & 415888 & 103972 & -21912 & -5478 & 496 & 124 & -4 & -1 & 0 \\
0 & 6350400 & -1587600 & -9203616 & 2300904 & 3247684 & -811921 & -415888 & 103972 & 21912 & -5478 & -496 & 124 & 4 & -1 & 0 \\
0 & 5080320 & 1016064 & -7477200 & -1495440 & 2759240 & 551848 & -384725 & -76945 & 22935 & 4587 & -575 & -115 & 5 & 1 & 0 \\
0 & -5080320 & 1016064 & 7477200 & -1495440 & -2759240 & 551848 & 384725 & -76945 & -22935 & 4587 & 575 & -115 & -5 & 1 & 0 \\
0 & -4233600 & -705600 & 6282744 & 1047124 & -2374086 & -395681 & 346632 & 57772 & -22308 & -3718 & 624 & 104 & -6 & -1 & 0 \\
0 & 4233600 & -705600 & -6282744 & 1047124 & 2374086 & -395681 & -346632 & 57772 & 22308 & -3718 & -624 & 104 & 6 & -1 & 0 \\
0 & 3628800 & 518400 & -5411952 & -773136 & 2074072 & 296296 & -311311 & -44473 & 21021 & 3003 & -637 & -91 & 7 & 1 & 0 \\
0 & -3628800 & 518400 & 5411952 & -773136 & -2074072 & 296296 & 311311 & -44473 & -21021 & 3003 & 637 & -91 & -7 & 1 & 0 \\
0 & -25401600 & 0 & 38402064 & 0 & -15291640 & 0 & 2475473 & 0 & -191620 & 0 & 7462 & 0 & -140 & 0 & 1
\end{pmatrix}$$

Winograd convolution with modulus(4001), F(14x14,3x3):

$$y_{4001} = g \circledast d \pmod{4001} = A_{4001}^T \Big[[G_{4001} g G_{4001}^T] \odot [B_{4001}^T d B_{4001}] \Big] A_{4001}$$

$$A_{4001}^T = \begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
0 & 1 & -1 & 2 & -2 & 3 & -3 & 4 & -4 & 5 & -5 & 6 & -6 & 7 & -7 & 0 \\
0 & 1 & 1 & 4 & 4 & 9 & 9 & 16 & 16 & 25 & 25 & 36 & 36 & 49 & 49 & 0 \\
0 & 1 & -1 & 8 & -8 & 27 & -27 & 64 & -64 & 125 & -125 & 216 & -216 & 343 & -343 & 0 \\
0 & 1 & 1 & 16 & 16 & 81 & 81 & 256 & 256 & 625 & 625 & 1296 & 1296 & -1600 & -1600 & 0 \\
0 & 1 & -1 & 32 & -32 & 243 & -243 & 1024 & -1024 & -876 & 876 & -226 & 226 & 803 & -803 & 0 \\
0 & 1 & 1 & 64 & 64 & 729 & 729 & 95 & 95 & -379 & -379 & -1356 & -1356 & 1620 & 1620 & 0 \\
0 & 1 & -1 & 128 & -128 & -1814 & 1814 & 380 & -380 & -1895 & 1895 & -134 & 134 & -663 & 663 & 0 \\
0 & 1 & 1 & 256 & 256 & -1441 & -1441 & 1520 & 1520 & -1473 & -1473 & -804 & -804 & -640 & -640 & 0 \\
0 & 1 & -1 & 512 & -512 & -322 & 322 & -1922 & 1922 & 637 & -637 & -823 & 823 & -479 & 479 & 0 \\
0 & 1 & 1 & 1024 & 1024 & -966 & -966 & 314 & 314 & -816 & -816 & -937 & -937 & 648 & 648 & 0 \\
0 & 1 & -1 & -1953 & 1953 & 1103 & -1103 & 1256 & -1256 & -79 & 79 & -1621 & 1621 & 535 & -535 & 0 \\
0 & 1 & 1 & 95 & 95 & -692 & -692 & 1023 & 1023 & -395 & -395 & -1724 & -1724 & -256 & -256 & 0 \\
0 & 1 & -1 & 190 & -190 & 1925 & -1925 & 91 & -91 & -1975 & 1975 & 1659 & -1659 & -1792 & 1792 & 1
\end{pmatrix}$$

$$G_{4001} = \begin{pmatrix}
422 & 0 & 0 \\
-631 & -631 & -631 \\
-631 & 631 & -631 \\
913 & 1826 & -349 \\
913 & -1826 & -349 \\
-1544 & -631 & -1893 \\
-1544 & 631 & -1893 \\
166 & 664 & -1345 \\
166 & -664 & -1345 \\
-1959 & -1793 & -963 \\
-1959 & 1793 & -963 \\
1853 & -885 & -1309 \\
1853 & 885 & -1309 \\
-725 & -1074 & 484 \\
-725 & 1074 & 484 \\
0 & 0 & 1
\end{pmatrix}$$

$$B_{4001}^T = \begin{pmatrix}
-749 & 0 & -466 & 0 & -182 & 0 & 1146 & 0 & -428 & 0 & 540 & 0 & 140 & 0 & -1 & 0 \\
0 & -749 & -749 & -1215 & -1215 & -1397 & -1397 & -251 & -251 & -679 & -679 & -139 & -139 & 1 & 1 & 0 \\
0 & 749 & -749 & 1215 & -1215 & 1397 & -1397 & 251 & -251 & 679 & -679 & 139 & -139 & -1 & 1 & 0 \\
0 & -1626 & -813 & 1827 & -1087 & 1548 & 774 & -186 & -93 & -1833 & 1084 & 272 & 136 & -2 & -1 & 0 \\
0 & 1626 & -813 & -1827 & -1087 & -1548 & 774 & 186 & -93 & 1833 & 1084 & -272 & 136 & 2 & -1 & 0 \\
0 & 1084 & 1695 & -924 & -308 & -1497 & -499 & -1118 & 961 & -1156 & -1719 & -393 & -131 & 3 & 1 & 0 \\
0 & -1084 & 1695 & 924 & -308 & 1497 & -499 & 1118 & 961 & 1156 & -1719 & 393 & -131 & -3 & 1 & 0 \\
0 & -813 & 797 & 1316 & 329 & 1128 & 282 & -216 & -54 & -1907 & -1477 & 496 & 124 & -4 & -1 & 0 \\
0 & 813 & 797 & -1316 & 329 & -1128 & 282 & 216 & -54 & 1907 & -1477 & -496 & 124 & 4 & -1 & 0 \\
0 & -950 & -190 & 669 & 934 & -1450 & -290 & -629 & -926 & -1071 & 586 & -575 & -115 & 5 & 1 & 0 \\
0 & 950 & -190 & -669 & 934 & 1450 & -290 & 629 & -926 & 1071 & 586 & 575 & -115 & -5 & 1 & 0 \\
0 & -542 & -1424 & 1174 & -1138 & -1493 & 418 & -1455 & 1758 & 1698 & 283 & 624 & 104 & -6 & -1 & 0 \\
0 & 542 & -1424 & -1174 & -1138 & 1493 & 418 & 1455 & 1758 & -1698 & 283 & -624 & 104 & 6 & -1 & 0 \\
0 & -107 & -1730 & 1401 & -943 & 1554 & 222 & 767 & -462 & 1016 & -998 & -637 & -91 & 7 & 1 & 0 \\
0 & 107 & -1730 & -1401 & -943 & -1554 & 222 & -767 & -462 & -1016 & -998 & 637 & -91 & -7 & 1 & 0 \\
0 & 749 & 0 & 466 & 0 & 182 & 0 & -1146 & 0 & 428 & 0 & -540 & 0 & -140 & 0 & 1
\end{pmatrix}$$

Winograd convolution with modulus(4331), F(14x14,3x3):

$$y_{4331} = g \circledast d \pmod{4331} = A_{4331}^T \Big[[G_{4331} g G_{4331}^T] \odot [B_{4331}^T d B_{4331}] \Big] A_{4331}$$

$$A_{4331}^T = \begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
0 & 1 & -1 & 2 & -2 & 3 & -3 & 4 & -4 & 5 & -5 & 6 & -6 & 7 & -7 & 0 \\
0 & 1 & 1 & 4 & 4 & 9 & 9 & 16 & 16 & 25 & 25 & 36 & 36 & 49 & 49 & 0 \\
0 & 1 & -1 & 8 & -8 & 27 & -27 & 64 & -64 & 125 & -125 & 216 & -216 & 343 & -343 & 0 \\
0 & 1 & 1 & 16 & 16 & 81 & 81 & 256 & 256 & 625 & 625 & 1296 & 1296 & -1930 & -1930 & 0 \\
0 & 1 & -1 & 32 & -32 & 243 & -243 & 1024 & -1024 & -1206 & 1206 & -886 & 886 & -517 & 517 & 0 \\
0 & 1 & 1 & 64 & 64 & 729 & 729 & -235 & -235 & -1699 & -1699 & -985 & -985 & 712 & 712 & 0 \\
0 & 1 & -1 & 128 & -128 & -2144 & 2144 & -940 & 940 & 167 & -167 & -1579 & 1579 & 653 & -653 & 0 \\
0 & 1 & 1 & 256 & 256 & -2101 & -2101 & 571 & 571 & 835 & 835 & -812 & -812 & 240 & 240 & 0 \\
0 & 1 & -1 & 512 & -512 & -1972 & 1972 & -2047 & 2047 & -156 & 156 & -541 & 541 & 1680 & -1680 & 0 \\
0 & 1 & 1 & 1024 & 1024 & -1585 & -1585 & 474 & 474 & -780 & -780 & 1085 & 1085 & -1233 & -1233 & 0 \\
0 & 1 & -1 & 2048 & -2048 & -424 & 424 & 1896 & -1896 & 431 & -431 & -2152 & 2152 & 31 & -31 & 0 \\
0 & 1 & 1 & -235 & -235 & -1272 & -1272 & -1078 & -1078 & 2155 & 2155 & 81 & 81 & 217 & 217 & 0 \\
0 & 1 & -1 & -470 & 470 & 515 & -515 & 19 & -19 & 2113 & -2113 & 486 & -486 & 1519 & -1519 & 1
\end{pmatrix}$$

$$G_{4331} = \begin{pmatrix}
-851 & 0 & 0 \\
-1286 & -1286 & -1286 \\
-1286 & 1286 & -1286 \\
2030 & -271 & -542 \\
2030 & 271 & -542 \\
1015 & -1286 & 473 \\
1015 & 1286 & 473 \\
1944 & -886 & 787 \\
1944 & 886 & 787 \\
486 & -1901 & -843 \\
486 & 1901 & -843 \\
-1591 & -884 & -973 \\
-1591 & 884 & -973 \\
-423 & 1370 & 928 \\
-423 & -1370 & 928 \\
0 & 0 & 1
\end{pmatrix}$$

$$B_{4331}^T = \begin{pmatrix}
285 & 0 & 913 & 0 & -1121 & 0 & 1859 & 0 & 1056 & 0 & 1200 & 0 & 140 & 0 & -1 & 0 \\
0 & 285 & 285 & 1198 & 1198 & 77 & 77 & 1936 & 1936 & -1339 & -1339 & -139 & -139 & 1 & 1 & 0 \\
0 & -285 & 285 & -1198 & 1198 & -77 & 77 & -1936 & 1936 & 1339 & -1339 & 139 & -139 & -1 & 1 & 0 \\
0 & 2023 & -1154 & 1132 & 566 & -1322 & -661 & -1260 & -630 & -843 & 1744 & 272 & 136 & -2 & -1 & 0 \\
0 & -2023 & -1154 & -1132 & 566 & 1322 & -661 & 1260 & -630 & 843 & 1744 & -272 & 136 & 2 & -1 & 0 \\
0 & 95 & -1412 & -1610 & 907 & -1515 & -505 & 1895 & -812 & 1525 & 1952 & -393 & -131 & 3 & 1 & 0 \\
0 & -95 & -1412 & 1610 & 907 & 1515 & -505 & -1895 & -812 & -1525 & 1952 & 393 & -131 & -3 & 1 & 0 \\
0 & -1154 & 1877 & 241 & 1143 & 566 & -2024 & 112 & 28 & -257 & -1147 & 496 & 124 & -4 & -1 & 0 \\
0 & 1154 & 1877 & -241 & 1143 & -566 & -2024 & -112 & 28 & 257 & -1147 & -496 & 124 & 4 & -1 & 0 \\
0 & 57 & -1721 & -1894 & -1245 & 393 & 1811 & 734 & 1013 & 1280 & 256 & -575 & -115 & 5 & 1 & 0 \\
0 & -57 & -1721 & 1894 & -1245 & -393 & 1811 & -734 & 1013 & -1280 & 256 & 575 & -115 & -5 & 1 & 0 \\
0 & 2118 & 353 & -1537 & -978 & -698 & -1560 & 152 & 1469 & -653 & 613 & 624 & 104 & -6 & -1 & 0 \\
0 & -2118 & 353 & 1537 & -978 & 698 & -1560 & -152 & 1469 & 653 & 613 & -624 & 104 & 6 & -1 & 0 \\
0 & -578 & -1320 & 1798 & 2113 & -477 & 1788 & 521 & -1163 & -634 & -1328 & -637 & -91 & 7 & 1 & 0 \\
0 & 578 & -1320 & -1798 & 2113 & 477 & 1788 & -521 & -1163 & 634 & -1328 & 637 & -91 & -7 & 1 & 0 \\
0 & -285 & 0 & -913 & 0 & 1121 & 0 & -1859 & 0 & -1056 & 0 & -1200 & 0 & -140 & 0 & 1
\end{pmatrix}$$

1.5 Winograd convolution F(10x10,7x7):

$y = g \circledast d = A^T \left[[GgG^T] \odot [B^T dB] \right] A$

$$A^T = \begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
0 & 1 & -1 & 2 & -2 & 3 & -3 & 4 & -4 & 5 & -5 & 6 & -6 & 7 & -7 & 0 \\
0 & 1 & 1 & 4 & 4 & 9 & 9 & 16 & 16 & 25 & 25 & 36 & 36 & 49 & 49 & 0 \\
0 & 1 & -1 & 8 & -8 & 27 & -27 & 64 & -64 & 125 & -125 & 216 & -216 & 343 & -343 & 0 \\
0 & 1 & 1 & 16 & 16 & 81 & 81 & 256 & 256 & 625 & 625 & 1296 & 1296 & 2401 & 2401 & 0 \\
0 & 1 & -1 & 32 & -32 & 243 & -243 & 1024 & -1024 & 3125 & -3125 & 7776 & -7776 & 16807 & -16807 & 0 \\
0 & 1 & 1 & 64 & 64 & 729 & 729 & 4096 & 4096 & 15625 & 15625 & 46656 & 46656 & 117649 & 117649 & 0 \\
0 & 1 & -1 & 128 & -128 & 2187 & -2187 & 16384 & -16384 & 78125 & -78125 & 279936 & -279936 & 823543 & -823543 & 0 \\
0 & 1 & 1 & 256 & 256 & 6561 & 6561 & 65536 & 65536 & 390625 & 390625 & 1679616 & 1679616 & 5764801 & 5764801 & 0 \\
0 & 1 & -1 & 512 & -512 & 19683 & -19683 & 262144 & -262144 & 1953125 & -1953125 & 10077696 & -10077696 & 40353607 & -40353607 & 1
\end{pmatrix}$$

$$G = \begin{pmatrix}
\frac{1}{25401600} & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{29030400} & \frac{1}{29030400} & \frac{1}{29030400} & \frac{1}{29030400} & \frac{1}{29030400} & \frac{1}{29030400} & \frac{1}{29030400} \\
\frac{1}{29030400} & \frac{-1}{29030400} & \frac{1}{29030400} & \frac{-1}{29030400} & \frac{1}{29030400} & \frac{-1}{29030400} & \frac{1}{29030400} \\
\frac{1}{43545600} & \frac{1}{21772800} & \frac{1}{10886400} & \frac{1}{5443200} & \frac{1}{2721600} & \frac{1}{1360800} & \frac{1}{680400} \\
\frac{1}{43545600} & \frac{-1}{21772800} & \frac{1}{10886400} & \frac{-1}{5443200} & \frac{1}{2721600} & \frac{-1}{1360800} & \frac{1}{680400} \\
\frac{1}{87091200} & \frac{1}{29030400} & \frac{1}{9676800} & \frac{1}{3225600} & \frac{1}{1075200} & \frac{1}{358400} & \frac{3}{358400} \\
\frac{1}{87091200} & \frac{-1}{29030400} & \frac{1}{9676800} & \frac{-1}{3225600} & \frac{1}{1075200} & \frac{-1}{358400} & \frac{3}{358400} \\
\frac{1}{239500800} & \frac{1}{59875200} & \frac{1}{14968800} & \frac{1}{3742200} & \frac{1}{935550} & \frac{2}{467775} & \frac{8}{467775} \\
\frac{1}{239500800} & \frac{-1}{59875200} & \frac{1}{14968800} & \frac{-1}{3742200} & \frac{1}{935550} & \frac{-2}{467775} & \frac{8}{467775} \\
\frac{1}{958003200} & \frac{1}{191600640} & \frac{1}{38320128} & \frac{5}{38320128} & \frac{25}{38320128} & \frac{125}{38320128} & \frac{625}{38320128} \\
\frac{1}{958003200} & \frac{-1}{191600640} & \frac{1}{38320128} & \frac{-5}{38320128} & \frac{25}{38320128} & \frac{-125}{38320128} & \frac{625}{38320128} \\
\frac{1}{6227020800} & \frac{1}{1037836800} & \frac{1}{172972800} & \frac{1}{28828800} & \frac{1}{4804800} & \frac{1}{800800} & \frac{3}{400400} \\
\frac{1}{6227020800} & \frac{-1}{1037836800} & \frac{1}{172972800} & \frac{-1}{28828800} & \frac{1}{4804800} & \frac{-1}{800800} & \frac{3}{400400} \\
\frac{1}{87178291200} & \frac{1}{12454041600} & \frac{1}{1779148800} & \frac{7}{1779148800} & \frac{49}{1779148800} & \frac{343}{1779148800} & \frac{2401}{1779148800} \\
\frac{1}{87178291200} & \frac{-1}{12454041600} & \frac{1}{1779148800} & \frac{-7}{1779148800} & \frac{49}{1779148800} & \frac{-343}{1779148800} & \frac{2401}{1779148800} \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix} = \frac{1}{87178291200} \begin{pmatrix}
3432 & 0 & 0 & 0 & 0 & 0 & 0 \\
3003 & 3003 & 3003 & 3003 & 3003 & 3003 & 3003 \\
3003 & -3003 & 3003 & -3003 & 3003 & -3003 & 3003 \\
2002 & 4004 & 8008 & 16016 & 32032 & 64064 & 128128 \\
2002 & -4004 & 8008 & -16016 & 32032 & -64064 & 128128 \\
1001 & 3003 & 9009 & 27027 & 81081 & 243243 & 729729 \\
1001 & -3003 & 9009 & -27027 & 81081 & -243243 & 729729 \\
364 & 1456 & 5824 & 23296 & 93184 & 372736 & 1490944 \\
364 & -1456 & 5824 & -23296 & 93184 & -372736 & 1490944 \\
91 & 455 & 2275 & 11375 & 56875 & 284375 & 1421875 \\
91 & -455 & 2275 & -11375 & 56875 & -284375 & 1421875 \\
14 & 84 & 504 & 3024 & 18144 & 108864 & 653184 \\
14 & -84 & 504 & -3024 & 18144 & -108864 & 653184 \\
1 & 7 & 49 & 343 & 2401 & 16807 & 117649 \\
1 & -7 & 49 & -343 & 2401 & -16807 & 117649 \\
0 & 0 & 0 & 0 & 0 & 0 & 87178291200
\end{pmatrix}$$

$$B^T = \begin{pmatrix}
25401600 & 0 & -38402064 & 0 & 15291640 & 0 & -2475473 & 0 & 191620 & 0 & -7462 & 0 & 140 & 0 & -1 & 0 \\
0 & 25401600 & 25401600 & -13000464 & -13000464 & 2291176 & 2291176 & -184297 & -184297 & 7323 & 7323 & -139 & -139 & 1 & 1 & 0 \\
0 & -25401600 & 25401600 & 13000464 & -13000464 & -2291176 & 2291176 & 184297 & -184297 & -7323 & 7323 & 139 & -139 & -1 & 1 & 0 \\
0 & -12700800 & -6350400 & 16025832 & 8012916 & -3639362 & -1819681 & 327896 & 163948 & -13836 & -6918 & 272 & 136 & -2 & -1 & 0 \\
0 & 12700800 & -6350400 & -16025832 & 8012916 & 3639362 & -1819681 & -327896 & 163948 & 13836 & -6918 & -272 & 136 & 2 & -1 & 0 \\
0 & 8467200 & 2822400 & -11859888 & -3953296 & 3779448 & 1259816 & -405219 & -135073 & 18849 & 6283 & -393 & -131 & 3 & 1 & 0 \\
0 & -8467200 & 2822400 & 11859888 & -3953296 & -3779448 & 1259816 & 405219 & -135073 & -18849 & 6283 & 393 & -131 & -3 & 1 & 0 \\
0 & -6350400 & -1587600 & 9203616 & 2300904 & -3247684 & -811921 & 415888 & 103972 & -21912 & -5478 & 496 & 124 & -4 & -1 & 0 \\
0 & 6350400 & -1587600 & -9203616 & 2300904 & 3247684 & -811921 & -415888 & 103972 & 21912 & -5478 & -496 & 124 & 4 & -1 & 0 \\
0 & 5080320 & 1016064 & -7477200 & -1495440 & 2759240 & 551848 & -384725 & -76945 & 22935 & 4587 & -575 & -115 & 5 & 1 & 0 \\
0 & -5080320 & 1016064 & 7477200 & -1495440 & -2759240 & 551848 & 384725 & -76945 & -22935 & 4587 & 575 & -115 & -5 & 1 & 0 \\
0 & -4233600 & -705600 & 6282744 & 1047124 & -2374086 & -395681 & 346632 & 57772 & -22308 & -3718 & 624 & 104 & -6 & -1 & 0 \\
0 & 4233600 & -705600 & -6282744 & 1047124 & 2374086 & -395681 & -346632 & 57772 & 22308 & -3718 & -624 & 104 & 6 & -1 & 0 \\
0 & 3628800 & 518400 & -5411952 & -773136 & 2074072 & 296296 & -311311 & -44473 & 21021 & 3003 & -637 & -91 & 7 & 1 & 0 \\
0 & -3628800 & 518400 & 5411952 & -773136 & -2074072 & 296296 & 311311 & -44473 & -21021 & 3003 & 637 & -91 & -7 & 1 & 0 \\
0 & -25401600 & 0 & 38402064 & 0 & -15291640 & 0 & 2475473 & 0 & -191620 & 0 & 7462 & 0 & -140 & 0 & 1
\end{pmatrix}$$

Winograd convolution with modulus(251), F(10x10,7x7):

$$y_{251} = g \circledast d \pmod{251} = A_{251}^T \Big[[G_{251} g G_{251}^T] \odot [B_{251}^T d B_{251}] \Big] A_{251}$$

$$A_{251}^T = \begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
0 & 1 & -1 & 2 & -2 & 3 & -3 & 4 & -4 & 5 & -5 & 6 & -6 & 7 & -7 & 0 \\
0 & 1 & 1 & 4 & 4 & 9 & 9 & 16 & 16 & 25 & 25 & 36 & 36 & 49 & 49 & 0 \\
0 & 1 & -1 & 8 & -8 & 27 & -27 & 64 & -64 & 125 & -125 & -35 & 35 & 92 & -92 & 0 \\
0 & 1 & 1 & 16 & 16 & 81 & 81 & 5 & 5 & 123 & 123 & 41 & 41 & -109 & -109 & 0 \\
0 & 1 & -1 & 32 & -32 & -8 & 8 & 20 & -20 & 113 & -113 & -5 & 5 & -10 & 10 & 0 \\
0 & 1 & 1 & 64 & 64 & -24 & -24 & 80 & 80 & 63 & 63 & -30 & -30 & -70 & -70 & 0 \\
0 & 1 & -1 & -123 & 123 & -72 & 72 & 69 & -69 & 64 & -64 & 71 & -71 & 12 & -12 & 0 \\
0 & 1 & 1 & 5 & 5 & 35 & 35 & 25 & 25 & 69 & 69 & -76 & -76 & 84 & 84 & 0 \\
0 & 1 & -1 & 10 & -10 & 105 & -105 & 100 & -100 & 94 & -94 & 46 & -46 & 86 & -86 & 1
\end{pmatrix}$$

$$G_{251} = \begin{pmatrix}
-32 & 0 & 0 & 0 & 0 & 0 & 0 \\
-28 & -28 & -28 & -28 & -28 & -28 & -28 \\
-28 & 28 & -28 & 28 & -28 & 28 & -28 \\
65 & -121 & 9 & 18 & 36 & 72 & -107 \\
65 & 121 & 9 & -18 & 36 & -72 & -107 \\
-93 & -28 & -84 & -1 & -3 & -9 & -27 \\
-93 & 28 & -84 & 1 & -3 & 9 & -27 \\
-11 & -44 & 75 & 49 & -55 & 31 & 124 \\
-11 & 44 & 75 & -49 & -55 & -31 & 124 \\
60 & 49 & -6 & -30 & 101 & 3 & 15 \\
60 & -49 & -6 & 30 & 101 & -3 & 15 \\
-68 & 94 & 62 & 121 & -27 & 89 & 32 \\
-68 & -94 & 62 & -121 & -27 & -89 & 32 \\
31 & -34 & 13 & 91 & -116 & -59 & 89 \\
31 & 34 & 13 & -91 & -116 & 59 & 89 \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

$$B_{251}^T = \begin{pmatrix}
-102 & 0 & -68 & 0 & -33 & 0 & -111 & 0 & 107 & 0 & 68 & 0 & -111 & 0 & -1 & 0 \\
0 & -102 & -102 & 81 & 81 & 48 & 48 & -63 & -63 & 44 & 44 & 112 & 112 & 1 & 1 & 0 \\
0 & 102 & -102 & -81 & 81 & -48 & 48 & 63 & -63 & -44 & 44 & -112 & 112 & -1 & 1 & 0 \\
0 & 51 & -100 & -16 & -8 & -113 & 69 & 90 & 45 & -31 & 110 & 21 & -115 & -2 & -1 & 0 \\
0 & -51 & -100 & 16 & -8 & 113 & 69 & -90 & 45 & 31 & 110 & -21 & -115 & 2 & -1 & 0 \\
0 & -34 & -95 & 113 & -46 & -110 & 47 & -105 & -35 & 24 & 8 & 109 & 120 & 3 & 1 & 0 \\
0 & 34 & -95 & -113 & -46 & 110 & 47 & 105 & -35 & -24 & 8 & -109 & 120 & -3 & 1 & 0 \\
0 & -100 & -25 & -52 & -13 & 5 & 64 & -19 & 58 & -75 & 44 & -6 & 124 & -4 & -1 & 0 \\
0 & 100 & -25 & 52 & -13 & -5 & 64 & 19 & 58 & 75 & 44 & 6 & 124 & 4 & -1 & 0 \\
0 & 80 & 16 & 90 & 18 & -3 & -101 & 58 & 112 & 94 & 69 & -73 & -115 & 5 & 1 & 0 \\
0 & -80 & 16 & -90 & 18 & 3 & -101 & -58 & 112 & -94 & 69 & 73 & -115 & -5 & 1 & 0 \\
0 & 17 & -39 & -37 & -48 & 123 & -105 & 1 & 42 & 31 & 47 & 122 & 104 & -6 & -1 & 0 \\
0 & -17 & -39 & 37 & -48 & -123 & -105 & -1 & 42 & -31 & 47 & -122 & 104 & 6 & -1 & 0 \\
0 & 93 & 85 & 110 & -56 & 59 & 116 & -71 & -46 & -63 & -9 & 116 & -91 & 7 & 1 & 0 \\
0 & -93 & 85 & -110 & -56 & -59 & 116 & 71 & -46 & 63 & -9 & -116 & -91 & -7 & 1 & 0 \\
0 & 102 & 0 & 68 & 0 & 33 & 0 & 111 & 0 & -107 & 0 & -68 & 0 & 111 & 0 & 1
\end{pmatrix}$$

Winograd convolution with modulus(241), F(10x10,7x7):

$$y_{241} = g \circledast d \pmod{241} = A_{241}^T \left[[G_{241} g G_{241}^T] \odot [B_{241}^T d B_{241}] \right] A_{241}$$

$$A_{241}^T = \begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
0 & 1 & -1 & 2 & -2 & 3 & -3 & 4 & -4 & 5 & -5 & 6 & -6 & 7 & -7 & 0 \\
0 & 1 & 1 & 4 & 4 & 9 & 9 & 16 & 16 & 25 & 25 & 36 & 36 & 49 & 49 & 0 \\
0 & 1 & -1 & 8 & -8 & 27 & -27 & 64 & -64 & -116 & 116 & -25 & 25 & 102 & -102 & 0 \\
0 & 1 & 1 & 16 & 16 & 81 & 81 & 15 & 15 & -98 & -98 & 91 & 91 & -9 & -9 & 0 \\
0 & 1 & -1 & 32 & -32 & 2 & -2 & 60 & -60 & -8 & 8 & 64 & -64 & -63 & 63 & 0 \\
0 & 1 & 1 & 64 & 64 & 6 & 6 & -1 & -1 & -40 & -40 & -98 & -98 & 41 & 41 & 0 \\
0 & 1 & -1 & -113 & 113 & 18 & -18 & -4 & 4 & 41 & -41 & -106 & 106 & 46 & -46 & 0 \\
0 & 1 & 1 & 15 & 15 & 54 & 54 & -16 & -16 & -36 & -36 & 87 & 87 & 81 & 81 & 0 \\
0 & 1 & -1 & 30 & -30 & -79 & 79 & -64 & 64 & 61 & -61 & 40 & -40 & 85 & -85 & 1
\end{pmatrix}$$

$$G_{241} = \begin{pmatrix}
47 & 0 & 0 & 0 & 0 & 0 & 0 \\
11 & 11 & 11 & 11 & 11 & 11 & 11 \\
11 & -11 & 11 & -11 & 11 & -11 & 11 \\
-73 & 95 & -51 & -102 & 37 & 74 & -93 \\
-73 & -95 & -51 & 102 & 37 & -74 & -93 \\
84 & 11 & 33 & 99 & 56 & -73 & 22 \\
84 & -11 & 33 & -99 & 56 & 73 & 22 \\
-79 & -75 & -59 & 5 & 20 & 80 & 79 \\
-79 & 75 & -59 & -5 & 20 & -80 & 79 \\
-80 & 82 & -72 & -119 & -113 & -83 & 67 \\
-80 & -82 & -72 & 119 & -113 & 83 & 67 \\
-105 & 93 & 76 & -26 & 85 & 28 & -73 \\
-105 & -93 & 76 & 26 & 85 & -28 & -73 \\
113 & 68 & -6 & -42 & -53 & 111 & 54 \\
113 & -68 & -6 & 42 & -53 & -111 & 54 \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

$$B_{241}^T = \begin{pmatrix}
-41 & 0 & 81 & 0 & -51 & 0 & 79 & 0 & 25 & 0 & 9 & 0 & -101 & 0 & -1 & 0 \\
0 & -41 & -41 & 40 & 40 & -11 & -11 & 68 & 68 & 93 & 93 & 102 & 102 & 1 & 1 & 0 \\
0 & 41 & -41 & -40 & 40 & 11 & -11 & -68 & 68 & -93 & 93 & -102 & 102 & -1 & 1 & 0 \\
0 & -100 & -50 & 55 & -93 & -21 & 110 & -105 & 68 & -99 & 71 & 31 & -105 & -2 & -1 & 0 \\
0 & 100 & -50 & -55 & -93 & 21 & 110 & 105 & 68 & 99 & 71 & -31 & -105 & 2 & -1 & 0 \\
0 & -94 & 49 & -37 & 68 & 86 & 109 & -98 & -113 & 51 & 17 & 89 & 110 & 3 & 1 & 0 \\
0 & 94 & 49 & 37 & 68 & -86 & 109 & 98 & -113 & -51 & 17 & -89 & 110 & -3 & 1 & 0 \\
0 & -50 & 108 & 67 & 77 & 32 & 8 & -78 & 101 & 19 & 65 & 14 & -117 & -4 & -1 & 0 \\
0 & 50 & 108 & -67 & 77 & -32 & 8 & 78 & 101 & -19 & 65 & -14 & -117 & 4 & -1 & 0 \\
0 & 40 & 8 & 66 & -35 & 31 & -42 & -89 & -66 & 40 & 8 & -93 & -115 & 5 & 1 & 0 \\
0 & -40 & 8 & -66 & -35 & -31 & -42 & 89 & -66 & -40 & 8 & 93 & -115 & -5 & 1 & 0 \\
0 & 47 & 48 & 115 & -21 & 5 & 41 & 74 & -68 & 105 & -103 & -99 & 104 & -6 & -1 & 0 \\
0 & -47 & 48 & -115 & -21 & -5 & 41 & -74 & -68 & -105 & -103 & 99 & 104 & 6 & -1 & 0 \\
0 & 63 & 9 & -56 & -8 & 26 & 107 & 61 & 112 & 54 & 111 & 86 & -91 & 7 & 1 & 0 \\
0 & -63 & 9 & 56 & -8 & -26 & 107 & -61 & 112 & -54 & 111 & -86 & -91 & -7 & 1 & 0 \\
0 & 41 & 0 & -81 & 0 & 51 & 0 & -79 & 0 & -25 & 0 & -9 & 0 & 101 & 0 & 1
\end{pmatrix}$$

Winograd convolution with modulus(239), F(10x10,7x7):

$$y_{239} = g \circledast d \pmod{239} = A_{239}^T \Big[[G_{239} g G_{239}^T] \odot [B_{239}^T d B_{239}] \Big] A_{239}$$

$$A_{239}^T = \begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
0 & 1 & -1 & 2 & -2 & 3 & -3 & 4 & -4 & 5 & -5 & 6 & -6 & 7 & -7 & 0 \\
0 & 1 & 1 & 4 & 4 & 9 & 9 & 16 & 16 & 25 & 25 & 36 & 36 & 49 & 49 & 0 \\
0 & 1 & -1 & 8 & -8 & 27 & -27 & 64 & -64 & -114 & 114 & -23 & 23 & 104 & -104 & 0 \\
0 & 1 & 1 & 16 & 16 & 81 & 81 & 17 & 17 & -92 & -92 & 101 & 101 & 11 & 11 & 0 \\
0 & 1 & -1 & 32 & -32 & 4 & -4 & 68 & -68 & 18 & -18 & -111 & 111 & 77 & -77 & 0 \\
0 & 1 & 1 & 64 & 64 & 12 & 12 & 33 & 33 & 90 & 90 & 51 & 51 & 61 & 61 & 0 \\
0 & 1 & -1 & -111 & 111 & 36 & -36 & -107 & 107 & -28 & 28 & 67 & -67 & -51 & 51 & 0 \\
0 & 1 & 1 & 17 & 17 & 108 & 108 & 50 & 50 & 99 & 99 & -76 & -76 & -118 & -118 & 0 \\
0 & 1 & -1 & 34 & -34 & 85 & -85 & -39 & 39 & 17 & -17 & 22 & -22 & -109 & 109 & 1
\end{pmatrix}$$

$$G_{239} = \begin{pmatrix}
-84 & 0 & 0 & 0 & 0 & 0 & 0 \\
46 & 46 & 46 & 46 & 46 & 46 & 46 \\
46 & -46 & 46 & -46 & 46 & -46 & 46 \\
-49 & -98 & 43 & 86 & -67 & 105 & -29 \\
-49 & 98 & 43 & -86 & -67 & -105 & -29 \\
95 & 46 & -101 & -64 & 47 & -98 & -55 \\
95 & -46 & -101 & 64 & 47 & 98 & -55 \\
78 & 73 & 53 & -27 & -108 & 46 & -55 \\
78 & -73 & 53 & 27 & -108 & -46 & -55 \\
-100 & -22 & -110 & -72 & 118 & 112 & 82 \\
-100 & 22 & -110 & 72 & 118 & -112 & 82 \\
3 & 18 & 108 & -69 & 64 & -94 & -86 \\
3 & -18 & 108 & 69 & 64 & 94 & -86 \\
-51 & -118 & -109 & -46 & -83 & -103 & -4 \\
-51 & 118 & -109 & 46 & -83 & 103 & -4 \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

$$B_{239}^T = \begin{pmatrix}
-37 & 0 & -22 & 0 & -58 & 0 & 89 & 0 & -58 & 0 & -53 & 0 & -99 & 0 & -1 & 0 \\
0 & -37 & -37 & -59 & -59 & -117 & -117 & -28 & -28 & -86 & -86 & 100 & 100 & 1 & 1 & 0 \\
0 & 37 & -37 & 59 & -59 & 117 & -117 & 28 & -28 & 86 & -86 & -100 & 100 & -1 & 1 & 0 \\
0 & -101 & 69 & -74 & -37 & -109 & 65 & -12 & -6 & 26 & 13 & 33 & -103 & -2 & -1 & 0 \\
0 & 101 & 69 & 74 & -37 & 109 & 65 & 12 & -6 & -26 & 13 & -33 & -103 & 2 & -1 & 0 \\
0 & -92 & 49 & 9 & 3 & -98 & 47 & -114 & -38 & -32 & 69 & 85 & 108 & 3 & 1 & 0 \\
0 & 92 & 49 & -9 & 3 & 98 & 47 & 114 & -38 & 32 & 69 & -85 & 108 & -3 & 1 & 0 \\
0 & 69 & 77 & -35 & 51 & 87 & -38 & 28 & 7 & 76 & 19 & 18 & -115 & -4 & -1 & 0 \\
0 & -69 & 77 & 35 & 51 & -87 & -38 & -28 & 7 & -76 & 19 & -18 & -115 & 4 & -1 & 0 \\
0 & -103 & 75 & -85 & -17 & -15 & -3 & 65 & 13 & -9 & 46 & -97 & -115 & 5 & 1 & 0 \\
0 & 103 & 75 & 85 & -17 & 15 & -3 & -65 & 13 & 9 & 46 & 97 & -115 & -5 & 1 & 0 \\
0 & 46 & -72 & -88 & 65 & -99 & 103 & 82 & -66 & -81 & 106 & -93 & 104 & -6 & -1 & 0 \\
0 & -46 & -72 & 88 & 65 & 99 & 103 & -82 & -66 & 81 & 106 & 93 & 104 & 6 & -1 & 0 \\
0 & 63 & 9 & -36 & 29 & 30 & -64 & 106 & -19 & -11 & -104 & 80 & -91 & 7 & 1 & 0 \\
0 & -63 & 9 & 36 & 29 & -30 & -64 & -106 & -19 & 11 & -104 & -80 & -91 & -7 & 1 & 0 \\
0 & 37 & 0 & 22 & 0 & 58 & 0 & -89 & 0 & 58 & 0 & 53 & 0 & 99 & 0 & 1
\end{pmatrix}$$

1.6 Winograd convolution F(10x10,7x7):

$y = g \circledast d = A^T \Big[[GgG^T] \odot [B^T dB] \Big] A$

$$A^T = \begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
0 & 1 & -1 & 2 & -2 & 3 & -3 & 4 & -4 & 5 & -5 & 6 & -6 & 7 & -7 & 0 \\
0 & 1 & 1 & 4 & 4 & 9 & 9 & 16 & 16 & 25 & 25 & 36 & 36 & 49 & 49 & 0 \\
0 & 1 & -1 & 8 & -8 & 27 & -27 & 64 & -64 & 125 & -125 & 216 & -216 & 343 & -343 & 0 \\
0 & 1 & 1 & 16 & 16 & 81 & 81 & 256 & 256 & 625 & 625 & 1296 & 1296 & 2401 & 2401 & 0 \\
0 & 1 & -1 & 32 & -32 & 243 & -243 & 1024 & -1024 & 3125 & -3125 & 7776 & -7776 & 16807 & -16807 & 0 \\
0 & 1 & 1 & 64 & 64 & 729 & 729 & 4096 & 4096 & 15625 & 15625 & 46656 & 46656 & 117649 & 117649 & 0 \\
0 & 1 & -1 & 128 & -128 & 2187 & -2187 & 16384 & -16384 & 78125 & -78125 & 279936 & -279936 & 823543 & -823543 & 0 \\
0 & 1 & 1 & 256 & 256 & 6561 & 6561 & 65536 & 65536 & 390625 & 390625 & 1679616 & 1679616 & 5764801 & 5764801 & 0 \\
0 & 1 & -1 & 512 & -512 & 19683 & -19683 & 262144 & -262144 & 1953125 & -1953125 & 10077696 & -10077696 & 40353607 & -40353607 & 1
\end{pmatrix}$$

$$G = \begin{pmatrix}
\frac{1}{25401600} & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{29030400} & \frac{1}{29030400} & \frac{1}{29030400} & \frac{1}{29030400} & \frac{1}{29030400} & \frac{1}{29030400} & \frac{1}{29030400} \\
\frac{1}{29030400} & \frac{-1}{29030400} & \frac{1}{29030400} & \frac{-1}{29030400} & \frac{1}{29030400} & \frac{-1}{29030400} & \frac{1}{29030400} \\
\frac{1}{43545600} & \frac{1}{21772800} & \frac{1}{10886400} & \frac{1}{5443200} & \frac{1}{2721600} & \frac{1}{1360800} & \frac{1}{680400} \\
\frac{1}{43545600} & \frac{-1}{21772800} & \frac{1}{10886400} & \frac{-1}{5443200} & \frac{1}{2721600} & \frac{-1}{1360800} & \frac{1}{680400} \\
\frac{1}{87091200} & \frac{1}{29030400} & \frac{1}{9676800} & \frac{1}{3225600} & \frac{1}{1075200} & \frac{1}{358400} & \frac{3}{358400} \\
\frac{1}{87091200} & \frac{-1}{29030400} & \frac{1}{9676800} & \frac{-1}{3225600} & \frac{1}{1075200} & \frac{-1}{358400} & \frac{3}{358400} \\
\frac{1}{239500800} & \frac{1}{59875200} & \frac{1}{14968800} & \frac{1}{3742200} & \frac{1}{935550} & \frac{2}{467775} & \frac{8}{467775} \\
\frac{1}{239500800} & \frac{-1}{59875200} & \frac{1}{14968800} & \frac{-1}{3742200} & \frac{1}{935550} & \frac{-2}{467775} & \frac{8}{467775} \\
\frac{1}{958003200} & \frac{1}{191600640} & \frac{1}{38320128} & \frac{5}{38320128} & \frac{25}{38320128} & \frac{125}{38320128} & \frac{625}{38320128} \\
\frac{1}{958003200} & \frac{-1}{191600640} & \frac{1}{38320128} & \frac{-5}{38320128} & \frac{25}{38320128} & \frac{-125}{38320128} & \frac{625}{38320128} \\
\frac{1}{6227020800} & \frac{1}{1037836800} & \frac{1}{172972800} & \frac{1}{28828800} & \frac{1}{4804800} & \frac{1}{800800} & \frac{3}{400400} \\
\frac{1}{6227020800} & \frac{-1}{1037836800} & \frac{1}{172972800} & \frac{-1}{28828800} & \frac{1}{4804800} & \frac{-1}{800800} & \frac{3}{400400} \\
\frac{1}{87178291200} & \frac{1}{12454041600} & \frac{1}{1779148800} & \frac{7}{1779148800} & \frac{49}{1779148800} & \frac{343}{1779148800} & \frac{2401}{1779148800} \\
\frac{1}{87178291200} & \frac{-1}{12454041600} & \frac{1}{1779148800} & \frac{-7}{1779148800} & \frac{49}{1779148800} & \frac{-343}{1779148800} & \frac{2401}{1779148800} \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix} = \frac{1}{87178291200} \begin{pmatrix}
3432 & 0 & 0 & 0 & 0 & 0 & 0 \\
3003 & 3003 & 3003 & 3003 & 3003 & 3003 & 3003 \\
3003 & -3003 & 3003 & -3003 & 3003 & -3003 & 3003 \\
2002 & 4004 & 8008 & 16016 & 32032 & 64064 & 128128 \\
2002 & -4004 & 8008 & -16016 & 32032 & -64064 & 128128 \\
1001 & 3003 & 9009 & 27027 & 81081 & 243243 & 729729 \\
1001 & -3003 & 9009 & -27027 & 81081 & -243243 & 729729 \\
364 & 1456 & 5824 & 23296 & 93184 & 372736 & 1490944 \\
364 & -1456 & 5824 & -23296 & 93184 & -372736 & 1490944 \\
91 & 455 & 2275 & 11375 & 56875 & 284375 & 1421875 \\
91 & -455 & 2275 & -11375 & 56875 & -284375 & 1421875 \\
14 & 84 & 504 & 3024 & 18144 & 108864 & 653184 \\
14 & -84 & 504 & -3024 & 18144 & -108864 & 653184 \\
1 & 7 & 49 & 343 & 2401 & 16807 & 117649 \\
1 & -7 & 49 & -343 & 2401 & -16807 & 117649 \\
0 & 0 & 0 & 0 & 0 & 0 & 87178291200
\end{pmatrix}$$

$$B^T = \begin{pmatrix}
25401600 & 0 & -38402064 & 0 & 15291640 & 0 & -2475473 & 0 & 191620 & 0 & -7462 & 0 & 140 & 0 & -1 & 0 \\
0 & 25401600 & 25401600 & -13000464 & -13000464 & 2291176 & 2291176 & -184297 & -184297 & 7323 & 7323 & -139 & -139 & 1 & 1 & 0 \\
0 & -25401600 & 25401600 & 13000464 & -13000464 & -2291176 & 2291176 & 184297 & -184297 & -7323 & 7323 & 139 & -139 & -1 & 1 & 0 \\
0 & -12700800 & -6350400 & 16025832 & 8012916 & -3639362 & -1819681 & 327896 & 163948 & -13836 & -6918 & 272 & 136 & -2 & -1 & 0 \\
0 & 12700800 & -6350400 & -16025832 & 8012916 & 3639362 & -1819681 & -327896 & 163948 & 13836 & -6918 & -272 & 136 & 2 & -1 & 0 \\
0 & 8467200 & 2822400 & -11859888 & -3953296 & 3779448 & 1259816 & -405219 & -135073 & 18849 & 6283 & -393 & -131 & 3 & 1 & 0 \\
0 & -8467200 & 2822400 & 11859888 & -3953296 & -3779448 & 1259816 & 405219 & -135073 & -18849 & 6283 & 393 & -131 & -3 & 1 & 0 \\
0 & -6350400 & -1587600 & 9203616 & 2300904 & -3247684 & -811921 & 415888 & 103972 & -21912 & -5478 & 496 & 124 & -4 & -1 & 0 \\
0 & 6350400 & -1587600 & -9203616 & 2300904 & 3247684 & -811921 & -415888 & 103972 & 21912 & -5478 & -496 & 124 & 4 & -1 & 0 \\
0 & 5080320 & 1016064 & -7477200 & -1495440 & 2759240 & 551848 & -384725 & -76945 & 22935 & 4587 & -575 & -115 & 5 & 1 & 0 \\
0 & -5080320 & 1016064 & 7477200 & -1495440 & -2759240 & 551848 & 384725 & -76945 & -22935 & 4587 & 575 & -115 & -5 & 1 & 0 \\
0 & -4233600 & -705600 & 6282744 & 1047124 & -2374086 & -395681 & 346632 & 57772 & -22308 & -3718 & 624 & 104 & -6 & -1 & 0 \\
0 & 4233600 & -705600 & -6282744 & 1047124 & 2374086 & -395681 & -346632 & 57772 & 22308 & -3718 & -624 & 104 & 6 & -1 & 0 \\
0 & 3628800 & 518400 & -5411952 & -773136 & 2074072 & 296296 & -311311 & -44473 & 21021 & 3003 & -637 & -91 & 7 & 1 & 0 \\
0 & -3628800 & 518400 & 5411952 & -773136 & -2074072 & 296296 & 311311 & -44473 & -21021 & 3003 & 637 & -91 & -7 & 1 & 0 \\
0 & -25401600 & 0 & 38402064 & 0 & -15291640 & 0 & 2475473 & 0 & -191620 & 0 & 7462 & 0 & -140 & 0 & 1
\end{pmatrix}$$

Winograd convolution with modulus(4001), F(10x10,7x7):

$$y_{4001} = g \circledast d \pmod{4001} = A_{4001}^T \Big[[G_{4001} g G_{4001}^T] \odot [B_{4001}^T d B_{4001}] \Big] A_{4001}$$

$$A_{4001}^T = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 2 & -2 & 3 & -3 & 4 & -4 & 5 & -5 & 6 & -6 & 7 & -7 & 0 \\ 0 & 1 & 1 & 4 & 4 & 9 & 9 & 16 & 16 & 25 & 25 & 36 & 36 & 49 & 49 & 0 \\ 0 & 1 & -1 & 8 & -8 & 27 & -27 & 64 & -64 & 125 & -125 & 216 & -216 & 343 & -343 & 0 \\ 0 & 1 & 1 & 16 & 16 & 81 & 81 & 256 & 256 & 625 & 625 & 1296 & 1296 & -1600 & -1600 & 0 \\ 0 & 1 & -1 & 32 & -32 & 243 & -243 & 1024 & -1024 & -876 & 876 & -226 & 226 & 803 & -803 & 0 \\ 0 & 1 & 1 & 64 & 64 & 729 & 729 & 95 & 95 & -379 & -379 & -1356 & -1356 & 1620 & 1620 & 0 \\ 0 & 1 & -1 & 128 & -128 & -1814 & 1814 & 380 & -380 & -1895 & 1895 & -134 & 134 & -663 & 663 & 0 \\ 0 & 1 & 1 & 256 & 256 & -1441 & -1441 & 1520 & 1520 & -1473 & -1473 & -804 & -804 & -640 & -640 & 0 \\ 0 & 1 & -1 & 512 & -512 & -322 & 322 & -1922 & 1922 & 637 & -637 & -823 & 823 & -479 & 479 & 1 \end{pmatrix}$$

$$G_{4001} = \begin{pmatrix} 422 & 0 & 0 & 0 & 0 & 0 & 0 \\ -631 & -631 & -631 & -631 & -631 & -631 & -631 \\ -631 & 631 & -631 & 631 & -631 & 631 & -631 \\ 913 & 1826 & -349 & -698 & -1396 & 1209 & -1583 \\ 913 & -1826 & -349 & 698 & -1396 & -1209 & -1583 \\ -1544 & -631 & -1893 & -1678 & -1033 & 902 & -1295 \\ -1544 & 631 & -1893 & 1678 & -1033 & -902 & -1295 \\ 166 & 664 & -1345 & -1379 & -1515 & 1942 & -234 \\ 166 & -664 & -1345 & 1379 & -1515 & -1942 & -234 \\ -1959 & -1793 & -963 & -814 & -69 & -345 & -1725 \\ -1959 & 1793 & -963 & 814 & -69 & 345 & -1725 \\ 1853 & -885 & -1309 & 148 & 888 & 1327 & -40 \\ 1853 & 885 & -1309 & -148 & 888 & -1327 & -40 \\ -725 & -1074 & 484 & -613 & -290 & 1971 & 1794 \\ -725 & 1074 & 484 & 613 & -290 & -1971 & 1794 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B_{4001}^T = \begin{pmatrix} -749 & 0 & -466 & 0 & -182 & 0 & 1146 & 0 & -428 & 0 & 540 & 0 & 140 & 0 & -1 & 0 \\ 0 & -749 & -749 & -1215 & -1215 & -1397 & -1397 & -251 & -251 & -679 & -679 & -139 & -139 & 1 & 1 & 0 \\ 0 & 749 & -749 & 1215 & -1215 & 1397 & -1397 & 251 & -251 & 679 & -679 & 139 & -139 & -1 & 1 & 0 \\ 0 & -1626 & -813 & 1827 & -1087 & 1548 & 774 & -186 & -93 & -1833 & 1084 & 272 & 136 & -2 & -1 & 0 \\ 0 & 1626 & -813 & -1827 & -1087 & -1548 & 774 & 186 & -93 & 1833 & 1084 & -272 & 136 & 2 & -1 & 0 \\ 0 & 1084 & 1695 & -924 & -308 & -1497 & -499 & -1118 & 961 & -1156 & -1719 & -393 & -131 & 3 & 1 & 0 \\ 0 & -1084 & 1695 & 924 & -308 & 1497 & -499 & 1118 & 961 & 1156 & -1719 & 393 & -131 & -3 & 1 & 0 \\ 0 & -813 & 797 & 1316 & 329 & 1128 & 282 & -216 & -54 & -1907 & -1477 & 496 & 124 & -4 & -1 & 0 \\ 0 & 813 & 797 & -1316 & 329 & -1128 & 282 & 216 & -54 & 1907 & -1477 & -496 & 124 & 4 & -1 & 0 \\ 0 & -950 & -190 & 669 & 934 & -1450 & -290 & -629 & -926 & -1071 & 586 & -575 & -115 & 5 & 1 & 0 \\ 0 & 950 & -190 & -669 & 934 & 1450 & -290 & 629 & -926 & 1071 & 586 & 575 & -115 & -5 & 1 & 0 \\ 0 & -542 & -1424 & 1174 & -1138 & -1493 & 418 & -1455 & 1758 & 1698 & 283 & 624 & 104 & -6 & -1 & 0 \\ 0 & 542 & -1424 & -1174 & -1138 & 1493 & 418 & 1455 & 1758 & -1698 & 283 & -624 & 104 & 6 & -1 & 0 \\ 0 & -107 & -1730 & 1401 & -943 & 1554 & 222 & 767 & -462 & 1016 & -998 & -637 & -91 & 7 & 1 & 0 \\ 0 & 107 & -1730 & -1401 & -943 & -1554 & 222 & -767 & -462 & -1016 & -998 & 637 & -91 & -7 & 1 & 0 \\ 0 & 749 & 0 & 466 & 0 & 182 & 0 & -1146 & 0 & 428 & 0 & -540 & 0 & -140 & 0 & 1 \end{pmatrix}$$

Winograd convolution with modulus(4331), F(10x10,7x7):

$$y_{4331} = g \circledast d \pmod{4331} = A_{4331}^T \left[[G_{4331} g G_{4331}^T] \odot [B_{4331}^T d B_{4331}] \right] A_{4331}$$

$$A_{4331}^T = \begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
0 & 1 & -1 & 2 & -2 & 3 & -3 & 4 & -4 & 5 & -5 & 6 & -6 & 7 & -7 & 0 \\
0 & 1 & 1 & 4 & 4 & 9 & 9 & 16 & 16 & 25 & 25 & 36 & 36 & 49 & 49 & 0 \\
0 & 1 & -1 & 8 & -8 & 27 & -27 & 64 & -64 & 125 & -125 & 216 & -216 & 343 & -343 & 0 \\
0 & 1 & 1 & 16 & 16 & 81 & 81 & 256 & 256 & 625 & 625 & 1296 & 1296 & -1930 & -1930 & 0 \\
0 & 1 & -1 & 32 & -32 & 243 & -243 & 1024 & -1024 & -1206 & 1206 & -886 & 886 & -517 & 517 & 0 \\
0 & 1 & 1 & 64 & 64 & 729 & 729 & -235 & -235 & -1699 & -1699 & -985 & -985 & 712 & 712 & 0 \\
0 & 1 & -1 & 128 & -128 & -2144 & 2144 & -940 & 940 & 167 & -167 & -1579 & 1579 & 653 & -653 & 0 \\
0 & 1 & 1 & 256 & 256 & -2101 & -2101 & 571 & 571 & 835 & 835 & -812 & -812 & 240 & 240 & 0 \\
0 & 1 & -1 & 512 & -512 & -1972 & 1972 & -2047 & 2047 & -156 & 156 & -541 & 541 & 1680 & -1680 & 1
\end{pmatrix}$$

$$G_{4331} = \begin{pmatrix}
-851 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1286 & -1286 & -1286 & -1286 & -1286 & -1286 & -1286 \\
-1286 & 1286 & -1286 & 1286 & -1286 & 1286 & -1286 \\
2030 & -271 & -542 & -1084 & 2163 & -5 & -10 \\
2030 & 271 & -542 & 1084 & 2163 & 5 & -10 \\
1015 & -1286 & 473 & 1419 & -74 & -222 & -666 \\
1015 & 1286 & 473 & -1419 & -74 & 222 & -666 \\
1944 & -886 & 787 & -1183 & -401 & -1604 & -2085 \\
1944 & 886 & 787 & 1183 & -401 & 1604 & -2085 \\
486 & -1901 & -843 & 116 & 580 & -1431 & 1507 \\
486 & 1901 & -843 & -116 & 580 & 1431 & 1507 \\
-1591 & -884 & -973 & -1507 & -380 & 2051 & -687 \\
-1591 & 884 & -973 & 1507 & -380 & -2051 & -687 \\
-423 & 1370 & 928 & 2165 & 2162 & 2141 & 1994 \\
-423 & -1370 & 928 & -2165 & 2162 & -2141 & 1994 \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

$$B_{4331}^T = \begin{pmatrix}
285 & 0 & 913 & 0 & -1121 & 0 & 1859 & 0 & 1056 & 0 & 1200 & 0 & 140 & 0 & -1 & 0 \\
0 & 285 & 285 & 1198 & 1198 & 77 & 77 & 1936 & 1936 & -1339 & -1339 & -139 & -139 & 1 & 1 & 0 \\
0 & -285 & 285 & -1198 & 1198 & -77 & 77 & -1936 & 1936 & 1339 & -1339 & 139 & -139 & -1 & 1 & 0 \\
0 & 2023 & -1154 & 1132 & 566 & -1322 & -661 & -1260 & -630 & -843 & 1744 & 272 & 136 & -2 & -1 & 0 \\
0 & -2023 & -1154 & -1132 & 566 & 1322 & -661 & 1260 & -630 & 843 & 1744 & -272 & 136 & 2 & -1 & 0 \\
0 & 95 & -1412 & -1610 & 907 & -1515 & -505 & 1895 & -812 & 1525 & 1952 & -393 & -131 & 3 & 1 & 0 \\
0 & -95 & -1412 & 1610 & 907 & 1515 & -505 & -1895 & -812 & -1525 & 1952 & 393 & -131 & -3 & 1 & 0 \\
0 & -1154 & 1877 & 241 & 1143 & 566 & -2024 & 112 & 28 & -257 & -1147 & 496 & 124 & -4 & -1 & 0 \\
0 & 1154 & 1877 & -241 & 1143 & -566 & -2024 & -112 & 28 & 257 & -1147 & -496 & 124 & 4 & -1 & 0 \\
0 & 57 & -1721 & -1894 & -1245 & 393 & 1811 & 734 & 1013 & 1280 & 256 & -575 & -115 & 5 & 1 & 0 \\
0 & -57 & -1721 & 1894 & -1245 & -393 & 1811 & -734 & 1013 & -1280 & 256 & 575 & -115 & -5 & 1 & 0 \\
0 & 2118 & 353 & -1537 & -978 & -698 & -1560 & 152 & 1469 & -653 & 613 & 624 & 104 & -6 & -1 & 0 \\
0 & -2118 & 353 & 1537 & -978 & 698 & -1560 & -152 & 1469 & 653 & 613 & -624 & 104 & 6 & -1 & 0 \\
0 & -578 & -1320 & 1798 & 2113 & -477 & 1788 & 521 & -1163 & -634 & -1328 & -637 & -91 & 7 & 1 & 0 \\
0 & 578 & -1320 & -1798 & 2113 & 477 & 1788 & -521 & -1163 & 634 & -1328 & 637 & -91 & -7 & 1 & 0 \\
0 & -285 & 0 & -913 & 0 & 1121 & 0 & -1859 & 0 & -1056 & 0 & -1200 & 0 & -140 & 0 & 1
\end{pmatrix}$$