

# Efficient Point Cloud Analysis Using Hilbert Curve

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## 1 More Ablation Study

In this part, we further measure the importance of Hilbert attention and Hilbert interpolation, which contribute most to the performance. ShapeNetPart [1] is used for ablation study.

### 1.1 Importance of Hilbert Attention

Since Hilbert attention is proposed to replace 3D convolution for handling grid data. Thus, we compare Hilbert Attention with 3D convolution at first. Specifically, we replace all Hilbert attention modules with 3D sparse convolution, and therefore VHF module and Hilbert pooling module are removed. Also, in Hilbert interpolation, the Bilinear kernel will be replaced by a Trilinear kernel. The new model is marked as Voxel-HilbertNet. The results in Table 1 show that if replacing Hilbert attention with 3D sparse convolution, the inference time will increase dramatically while the performance only has 0.1% gain. Also, Voxel-HilbertNet results with voxel size  $32^3$  tell that voxel resolution plays a very important role in the segmentation task. Although the inference speed is increased with the decrease of voxel size in Voxel-HilbertNet, the sacrifice in segmentation accuracy will not be acceptable.

The experiment further illustrates the importance of Hilbert attention, that is, **approximates the performance of 3D convolution while keeping the computational cost at a relatively low level such that we can use the data with a larger voxel size while having acceptable inference speed.**

### 1.2 Importance of Hilbert Interpolation

In this part, we make a further study of Hilbert interpolation, which is one of the main components for the success of HilbertNet. Hilbert interpolation outperforms traditional linear interpolation not only because of its locality preserving property but also because Hilbert curve has better *clustering property* [4].

**Definition 1 (Cluster)** *Given a  $D$ -dimensional query window, the points inside the window that are consecutively connected by a mapping is defined as a cluster.*

Table 1: Comparison of methods

Method	voxel size	Inference time	mIoU
3D-UNet [3]	$64^3$	347ms	84.2
PVCNN [2]	$32^3$	62.5ms	86.0
HilbertNet-L	$64^3$	<b>42.1ms</b>	85.8
HilbertNet-M	$64^3$	59.2ms	86.4
HilbertNet	$64^3$	91.6ms	87.1
Voxel-HilbertNet	$64^3$	198ms	<b>87.2</b>
Voxel-HilbertNet	$32^3$	78.4ms	86.3

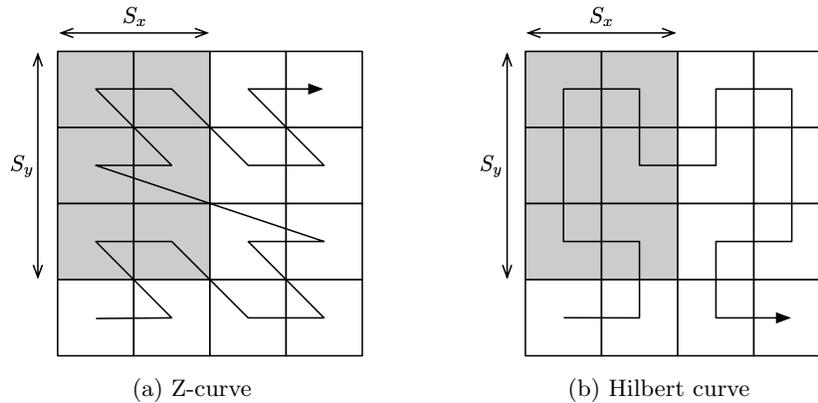


Fig. 1: The number of cluster for difference space-filling curves. For Z-curve, the points inside the query window are consecutively connected by 2 lines, while for the Hilbert curve, all points inside the query window are consecutively connected by 1 line.

For example, if applying a query window with length  $S_x$  and width  $S_y$  to Z-curve [5] and Hilbert curve (see Fig. 1), the number of clusters inside Z-curve is 2 while Hilbert curve only contains 1 cluster.

For a space-filling curve, the smaller number of clusters, the better clustering property. Therefore, we test the **average number of cluster** of reshape function, Z-curve, and Hilbert curve to find which one is the most favorable for data clustering. The results can be found in Fig. 2. Here we use a square query window to sweep a point array with size  $1024 \times 1024$  and the window size is increased from 2 to 32. It can be found that Hilbert curve constantly has the lowest number of clusters, which demonstrates its good clustering property.

## References

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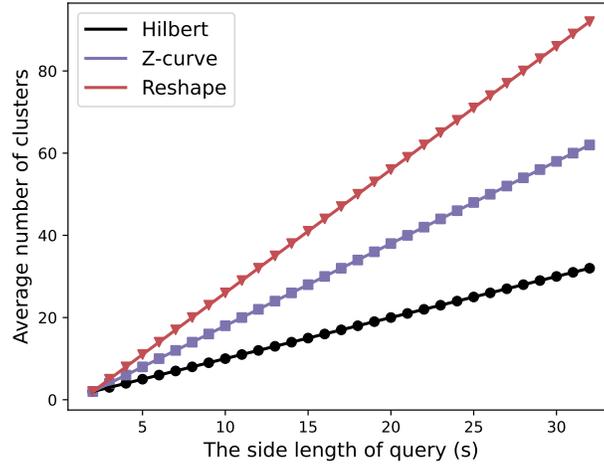


Fig. 2: Average number of clusters of different space-filling curves.

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