Supplementary Material: Kernel Relative-prototype Spectral Filtering for Few-shot Learning

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1 Appendix

1.1 Proof of Theorem 2

Suppose that: 1) $h(\gamma_{i,c}, \lambda_{l,c}) = \mathbb{I}_{(\gamma_{i,c} \geq \lambda_{l,c})}\gamma_{i,c}^{-1}$; 2) $\zeta = 1$; 3) $\lambda_{l,c} =$ constant for all l and c; 4) the map function ϕ_ω is identical. Eq. 17 is reduced to $d^2_{l,c}(\lambda, \boldsymbol{S}_c, \boldsymbol{q}_l) = \|(I - P_c P_c^T)(f_\theta(\boldsymbol{q}_l) - \boldsymbol{\mu}_c)\|^2$ with P_c the truncated matrix of W_c, where W_c is the eigenvector matrix of empirical covariance matrix C.

Proof.

$$\begin{aligned}
d^2_{l,c}(\lambda, \boldsymbol{S}_c, \boldsymbol{q}_l) &= \|\boldsymbol{\mu}_{l,c}(\lambda_c)\|^2 \\
&= \|\boldsymbol{\mu}_{l,c} - \sum_{i=1}^{n} h(\gamma_{i,c}, \lambda_{l,c})\gamma_{i,c}\langle\boldsymbol{\mu}_{l,c}, \boldsymbol{w}_{i,c}\rangle\boldsymbol{w}_{i,c}\|^2 \\
&= \|\boldsymbol{\mu}_{l,c} - \sum_{i=1}^{n} \mathbb{I}_{(\gamma_{i,c} \geq \lambda)}\langle\boldsymbol{\mu}_{l,c}, \boldsymbol{w}_{i,c}\rangle\boldsymbol{w}_{i,c}\|^2 \\
&= \|\boldsymbol{\mu}_{l,c} - \boldsymbol{W}_c G\langle\boldsymbol{W}_c^T, \boldsymbol{\mu}_{l,c}\rangle\|^2 \\
&= \|\boldsymbol{\mu}_{l,c} - \boldsymbol{W}_c G\boldsymbol{W}_c^T\boldsymbol{\mu}_{l,c}\|^2 \\
&= \|\boldsymbol{\mu}_{l,c} - \boldsymbol{P}_c\boldsymbol{P}_c^T\boldsymbol{\mu}_{l,c}\|^2 \\
&= \|(\boldsymbol{I} - \boldsymbol{P}_c\boldsymbol{P}_c^T)\boldsymbol{\mu}_{l,c}\|^2 \\
&= \|(\boldsymbol{I} - \boldsymbol{P}_c\boldsymbol{P}_c^T)(\phi_\omega(f_\theta(\boldsymbol{q}_l)) - \boldsymbol{\mu}_c)\|^2 \\
&= \|(\boldsymbol{I} - \boldsymbol{P}_c\boldsymbol{P}_c^T)(f_\theta(\boldsymbol{q}_l) - \boldsymbol{\mu}_c)\|^2
\end{aligned}, \tag{1}$$

where $\boldsymbol{W}_c = \{\boldsymbol{w}_{1,c}, \boldsymbol{w}_{2,c} ..., \boldsymbol{w}_{n,c}\}$ and $G = \mathrm{diag}(\mathbb{I}_{(\gamma_{1,c} \geq \lambda)}, \mathbb{I}_{(\gamma_{2,c} \geq \lambda)}, ..., \mathbb{I}_{(\gamma_{n,c} \geq \lambda)})$.