1 Proof of Theorem 2

Proof. First we make some notations. Suppose we have a grid of size (q, q). For a position (x, y) that $1 \leq x, y \leq q$, denote $UL(x, y)$ to be the generalized window with size $(q/2 + p_n, q/2 + p_n)$ whose upper left corner lies on (x, y), and $L(y)$ to be the set of generalized windows with size $(q/2 + p_n, q/2 + p_n)$ whose left side lies on y. For a generalized window M, denote $L(M), R(M), U(M), Lo(M)$ to be the position of left, right, upper and lower side respectively.

Considering two generalized windows M_1, M_2 with size (p_n, p_n) on the $q \times q$ grid. Without loss of generality, we assume $L(M_1) \leq L(M_2)$. Also we assume $U(M_1) \leq U(M_2)$, otherwise we can rotate the grid, and find the generalized window on the rotated grid that corresponds to the one on origin grid. Now we consider different cases.

1. If $x_2 - x_1 \leq q/2, y_2 - y_1 \leq q/2$, obviously $UL(x_1, y_1)$ satisfies.

2. If $x_2 - x_1 \leq q/2, y_2 - y_1 > q/2$, we have $\forall M \in L(y_2)$, $R(M) \geq y_1 + p_n$. So we only need to find x such that $Lo(UL(x, y_2)) \geq \max\{x_1 + p_n, x_2 + p_n\}$. Actually $x = x_1$ is enough, so choose the generalized window to be $UL(x_1, y_2)$.

3. If $x_2 - x_1 > q/2, y_2 - y_1 \leq q/2$, similarly, we can choose $UL(x_2, y_1)$.

4. If $x_2 - x_1 > q/2, y_2 - y_1 > q/2$, choose $UL(x_2, y_2)$.

Hence such generalized window exists for all cases.