Supplementary Material: Decomposing The Tangent of Occluding Boundaries According to Curvatures and Torsions

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1 Derivation for silhouette matching

Given a known 3D shape and the current pose parameter λ to be optimized, we denote the 2D boundary of the silhouette as $\hat{C}$. A convenient way to derive the gradient of the energy with respect to the pose parameter is to divide integral region into two parts, one R^i inside $\hat{C}$ and the other R^o outside it. We may then rewrite the energy as follows,

$$E(\lambda) = \int_{R^i} (1-B)^2 d\mathbf{x} + \int_{R^o} B^2 d\mathbf{x} \tag{1}$$

Note the added square evaluations here would not affect the result comparing to the absolute evaluations as the quantities are either 0 or 1. We can further simplify the equation (1) as

$$\begin{aligned} E(\lambda) &= \int_{R^i} (1-B)^2 d\mathbf{x} + \int_{\Omega} B^2 d\mathbf{x} - \int_{R^i} B^2 d\mathbf{x} \\ &= \int_{R_i} (1-2B) d\mathbf{x} + \int_{\Omega} B^2 d\mathbf{x} \end{aligned} \tag{2}$$

Following equation (4) in the reference [1] and changing the order by the triplet product formula, we can reach to equation (7) in the main paper.

2 Implementation details

2.1 Coefficient calculations

While we present the formulas under the implicit frame work, the inclusion of which does not imply the unique applicable exploitation. Other parametric representations would directly benefit from the theorem.

Denoting as ψ the implicit function representing the surface, we may write the curvature and torsion at the point X as follows,

$$\tau_r = \left\langle \frac{\nabla\psi}{\|\nabla\psi\|}, \frac{X}{\|X\|} \times (\nabla^2\psi(\frac{\nabla\psi\nabla\psi^T}{\|\nabla\psi\|^2} - \mathcal{I})X) \right\rangle \tag{3}$$

$$\kappa_r = \left\langle \left(\frac{\nabla\psi}{\|\nabla\psi\|} \cdot \nabla^2\psi(\mathcal{I} - \frac{\nabla\psi\nabla\psi^T}{\|\nabla\psi\|^2})X \right) \frac{\nabla\psi}{\|\nabla\psi\|} - \nabla^2\psi(\mathcal{I} - \frac{\nabla\psi\nabla\psi^T}{\|\nabla\psi\|^2})X,\ \frac{X}{\|X\|} \right\rangle \tag{4}$$

2.2 Algorithm

Since the experiments are designed to demonstrate the numerical feasibility of the theorem, we did not exploit sophisticated methods to find the initial point. On the other hand, the initial point X_0 is picked by querying triangle visibility from rendering libraries (e.g. OpenGL) and selecting the first edge whose shared triangles have different visibility. But in real applications, the initial point would be conveniently found by ray-casting from 2D images. We follow the algorithm below to extract occluding boundaries.

Algorithm 1: Boundary extraction for one curve

Data: X_0 satisfying the visibility condition, step size s
Result: $\{\mathcal{S}^*\}$
$\mathcal{S}^* \leftarrow \{X_0\}$;
$\mathbf{t}^* \leftarrow$ tangentCalculation(X_0);
$i \leftarrow 1$;
while *no loop closure* **do**
  $X_i \leftarrow X_{i-1} + s * \mathbf{t}^*$
  $\mathcal{S}^* \leftarrow \mathcal{S}^{**} \cup \{X_i\}$
  $\mathbf{t}^* \leftarrow$ tangentCalculation(X_i)
  $i \leftarrow i + 1$
end

2.3 Numerical details

The level set function ψ whose zero set represents the surface is defined on a regular voxel volume, $128 \times 128 \times 128$ in our case. In most scenarios the occluding points would not be on the grid points. As such, we need to interpolate entities evaluated on these points. We first compute normals, torsion, curvatures at grid points (rounding the component of X up/down), whereby second-order central differences are used for gradient and Hessian elements, then we tri-linearly interpolate interpolate them at X. Tangent vectors would be calculated with these interpolated values.

References

1. Dambreville, S., Sandhu, R., Yezzi, A., Tannenbaum, A.: Robust 3d pose estimation and efficient 2d region-based segmentation from a 3d shape prior. In: ECCV. pp. 169–182. Springer (2008)